OPRA: Optimal Relay Assignment for Capacity Maximization in Cooperative Networks

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***Abstract*—As promising as it is, cooperative communication has been proposed to increase the capacity of wireless networks through spatial diversity. Without requiring multiple antennas on the same device, spatial diversity is achieved by exploiting the antennas on other nodes, i.e., relay nodes, in the network. Therefore, the selection of relay nodes has a significant impact on the achieved total capacity. In this paper, we study the problem of relay assignment in cooperative networks, where multiple source-destination transmission pairs share the same set of relay nodes. Specifically, we propose a system model where a relay node can be shared by multiple source-destination pairs and present a corresponding formulation for the capacity calculation. Our objective is to find a relay assignment to maximize the total capacity of the network. As the main contribution, we develop an optimal relay assignment algorithm to solve this problem in polynomial time. We also show that our algorithm has several attractive properties.**

1. INTRODUCTION

Through cooperative relaying from other devices (generally called relay nodes), Cooperative Communication (CC) [6] has been shown to have the potential to increase the channel capacity between two wireless devices. The essence of CC is to exploit the nature of broadcast and the relaying capability of other nodes to achieve spatial diversity. Two primary CC modes have been commonly used, *Amplify-and-Forward* (AF) and *Decode-and-Forward* (DF), depending on how the relay node processes the received signal and transmits to the destination. In the AF mode, the relay node amplifies the signal received from the source node and forwards it to the destination node. In the DF mode, the relay node decodes the signal received from the source node and re-encodes it before forwarding it to the destination node. Although CC promises to increase the capacity, an improper selection of relay node can result in an even smaller capacity than that under direct transmission. Therefore, the assignment of relay nodes plays an critical role in the performance of CC [2, 3, 12, 16].

We consider the following scenario in this paper. In a wireless network, there are a number of source-destination pairs and many other wireless devices functioning as relay nodes. We are interested in finding a relay assignment for each source-destination pair, such that the total capacity of all pairs is maximized.

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This problem has many applications in practice. For example, in WiMAX networks (based on the IEEE 802.16 standard), relay stations have been introduced to relay traffic for subscriber stations in order to meet the growing demand for throughput and capacity [1]. In this context, all the subscriber stations are source nodes and they share the same destination node, the base station. In cellular networks, the demands for bandwidth-hungry multimedia and internet-based services have pushed the carrier companies to develop innovative network solutions with low cost. To this end, multihop cellular network (MCN) [7, 10] has emerged as an efficient and promising solution. In MCN, other mobile stations can relay the packets for the source mobile station to the destination mobile station if it is within the same cell as the source mobile station or to the base station otherwise.

Our main contributions can be summarized as follows:

- We study the relay assignment problem in cooperative networks, which seeks for a relay assignment for each source-destination pair so as to maximize the total capacity of all pairs.
- We consider a system model which allows a relay node to be shared by multiple source nodes. It is more practical compared with the model in [12], which restricts each relay to be assigned to only one source-destination pair.
- We design a polynomial time algorithm to optimally solve the relay assignment problem. That is, our algorithm can find a relay assignment that gives the maximum total capacity among all the possible assignments.
- We show that our algorithm works regardless of which CC mode is used in the network. It is also independent of the relation between the number of source-destination pairs and that of the relay nodes. In addition, our algorithm can guarantee that the achieved capacity between each source-destination pair after the assignment is no less than that of direct transmission.

The remainder of this paper is organized as follows. In Section 2, we give a brief review of the related work in the literature. In Section 3, we describe the system model for the relay assignment problem studied in this paper. In Section 4, we then present a polynomial time optimal algorithm to solve the relay assignment. Section 5 illustrates the numerical results. Finally, we conclude this paper in Section 6.

2. Related Work

In this section, we give a brief review of the related work on cooperative communication (CC), with the focus on relay assignment.

As the pioneer in the development of CC, Van Der Meulen [8] first introduced a three-terminal cooperative channel model and bounded the capacity for different relaying strategies. Another fundamental work on CC was done by Cover and El Gammal in [5]. They gave an achievable lower bound on the capacity of the general relay channel.

In [2], Bletsas *et al.* proposed a novel scheme to select the best relay node for a single source-destination pair from a set of available relays. However, this can not be extended to a network consisting multiple source-destination pairs, which is the model studied in this paper.

Some efforts have been made on the relay assignment or relay selection problem in cooperative networks. In [3], Cai *et al.* studied the problem of relay selection and power allocation for AF wireless relay networks. They first considered the simple network with only one source-destination pair, and then extended it to the multiple-pair case. The proposed algorithm is an effective heuristic, but offers no performance guarantee. From a different perspective, Xu *et al.* [14] studied a similar problem but with a different objective, which is to minimize the total power consumption of the network. In [9], Ng and Yu jointly considered the relay node selection, cooperative communication and resource allocation for utility maximization in a cellular network. However, the algorithm is heuristic and not polynomial, as pointed out by Shi *et al.* [12]. In [11], Sharma *et al.* investigated a joint problem of relay node assignment and multi-hop flow routing, with the objective to maximize the minimum rate among a set of concurrent sessions. They formulated the problem as a mixed integer linear programming and solved it using a *branch-and-cut* framework, which produces a $(1-\epsilon)$-approximation algorithm.

The most closely related work was done by Shi *et al.* [12]. They studied the relay assignment problem in a network environment, such that the minimum capacity among all source-destination pairs is maximized. Following this work, Zhang *et al.* [15] considered the relay assignment problem with interference mitigation. In both models in [12] and [15], a relay node is restricted to be assigned to at most one source-destination pair. In contrast, our model in this paper is more practical in the sense that it allows multiple source nodes to share the same relay node. In addition, different from [12], our objective is to maximize the total capacity of all pairs. Though Zhang *et al.* [15] had the same objective as ours, they only provided a heuristic algorithm.

3. System Model

In this paper, we consider a static ad hoc wireless network consisting of n source-destination pairs $\{s_1, d_1; s_2, d_2; \cdots ; s_n, d_n\}$ and a set $\mathcal{R} = \{r_1, r_2, \cdots , r_m\}$ of m relay nodes. We use $\mathcal{S} = \{s_1, s_2, \cdots , s_n\}$ to denote the set of source nodes and $\mathcal{D} = \{d_1, d_2, \cdots , d_n\}$ to denote the set of destination nodes. As in [12], we assume that orthogonal channels are available in the network (e.g. using OFDMA) to mitigate interference. We assume that each node is equipped with one single transceiver and can transmit/receive at a time. Let P_i^s denote the transmission power of source s_i and P_j^r denote the transmission power of relay node r_j. We model the channel propagation as follows. When node u transmits a signal to node v with power P_u, the signal-to-noise ratio (SNR) received at node v, denoted as SNR_{uv}, is

$$SNR_{uv} = \frac{P_u}{N_0 \cdot ||u, v||^{\alpha}}, \quad (1)$$

where N_0 is the abient noise, $||u, v||$ is the Euclidean distance between nodes u and v, and α is the path loss exponent which is between 2 and 4 in general, depending on the characteristics of the communication medium.

For the transmission model, we assume that each source-destination pair has an option to use cooperative communication (CC) with the help of a relay node. A recent work by Zhao *et al.* [16] showed that it is sufficient for a source-destination pair to choose the best relay node even when multiple are available to achieve full diversity. Therefore, it is reasonable to assume that each source-destination pair will either transmit directly or use CC with the help of only one relay node. Next, we present both the direct transmission model and cooperative communication model in details.

Direct Transmission: When the source node s directly transmits to the destination node d, the achievable capacity is

$$C_{DT}(s, d) = W \log_2(1 + SNR_{sd}), \quad (2)$$

where W is the bandwidth of the channel.

We use a well-known three-node example in Fig. 1 to describe the essence of CC. In this example, s is the source node that transmits information, d is the destination node that receives information and r is the relay node that both receives and transmits information to enhance the communication between the source and the destination. CC proceeds in a frame-by-frame fashion. Each frame is divided into two time slots. Source node s transmits data to the destination d in the first time slot. Due to the broadcast nature, relay node r can overhear this transmission. In the second time slot, r forwards the data to d using different techniques depending on different CC modes. There are two different CC modes, *Amplify-and-Forward* (AF) and *Decode-and-Forward* (DF) [6].

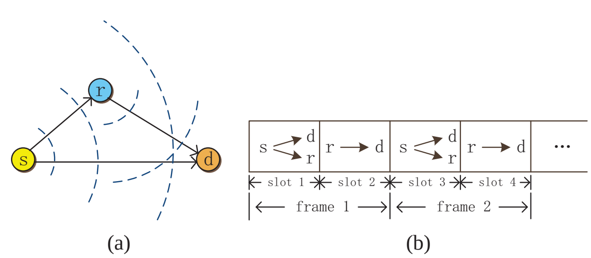


Fig. 1: A three-node example for CC

Amplify-and-Forward (AF): In the amplify-and-forward mode, the relay node amplifies the signal transmitted by the source node in the first time slot and then transmits the amplified signal to the destination in the second time slot. The achievable capacity from s to d is

$$C_{AF}(s,r,d)=\frac{W}{2}\log_2\left(1+SNR_{sd}+\frac{SNR_{sr}\cdot SNR_{rd}}{SNR_{sr}+SNR_{rd}+1}\right).$$

Decode-and-Forward (DF): In the decode-and-forward mode, the relay node decodes and estimates the signal transmitted by the source node in the first time slot and then transmits the estimated data to the destination in the second time slot. The achievable capacity from s to d is

$$C_{DF}(s,r,d)=\frac{W}{2}\min\{\log_2(1+SNR_{sr}), \log_2(1+SNR_{sd}+SNR_{rd})\}.$$

Since both C_{AF} and C_{DF} are functions of P_r, $||s,r||$ and $||r,d||$, it is unobvious whether using CC can benefit the transmission between a source-destination pair, in the sense that $C_{AF}(s,r,d) > C_{DT}(s,d)$ or $C_{DF}(s,r,d) > C_{DT}(s,d)$. The algorithm developed in this paper is independent of the CC mode. Therefore we use C_R to denote the achievable capacity under the used CC mode, i.e., $C_R = C_{AF}$ if AF is used and $C_R = C_{DF}$ if DF is used. We say that a relay node r_j is *assigned* to a source node s_i if r_j helps s_i to achieve cooperative communication from s_i to d_i. Let $\sigma : \mathcal{S} \to \mathcal{R}\cup\{\phi\}$ be a relay assignment. Here $\sigma(s_i) = \phi$ means that s_i transmits directly to d_i. Note that it is possible to have $\sigma(s_i) = \sigma(s_j)$ for $s_i \neq s_j$. This is a major difference from the model in [12], in which a relay node can be assigned to only one source node. Since we do not have such constraints, our model is more realistic than the model in [12].

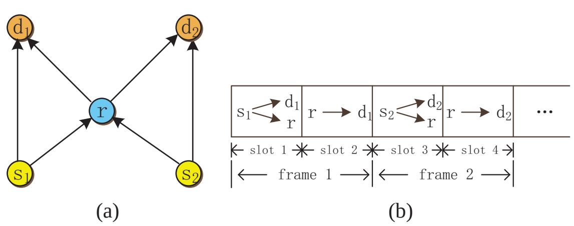


Fig. 2: Multi-source cooperative communication.

Now let us consider the case where the same relay node is assigned to multiple source nodes. We denote $\mathcal{S}(r_j)$ as the set of source nodes, which r_j is assigned to, i.e., $\mathcal{S}(r_j) = \{s_i|\sigma(s_i) = r_j\}$. In this case, we assume each relay node is shared fairly among all the source nodes employing that relay node. This can be achieved for example by using a reservation-based TDMA scheduling. The relay node serves each source node in a round-robin fashion. Each frame is dedicated to a single source node for CC. Each source node gets served every n_j frames, where $n_j = |\mathcal{S}(r_j)|$ is the number of source nodes that r_j is assigned to. Therefore, the average achievable capacity for each source node $s_i \in \mathcal{S}(r_j)$ is $\frac{C_R(s_i,\sigma(s_i),d_i)}{n_j}$. A simple example is shown in Fig. 2. For notational simplicity, we use $C(s_i, \sigma(s_i))$ to denote the achievable capacity when $\sigma(s_i)$ is assigned to s_i. Hereafter we also omit d_i in the capacity expression because it is clear that d_i and s_i have a one-one mapping relation. Thus we have

$$C(s_i,\sigma(s_i)) = \begin{cases} \frac{C_R(s_i,\sigma(s_i))}{n_j}, & \text{if } \sigma(s_i) \neq \phi, \\ C_{DT}(s_i), & \text{if } \sigma(s_i) = \phi. \end{cases} \quad (3)$$

Our objective in this paper is to find a relay assignment such that the total capacity is maximized. It is different from the problem in [12], of which the objective is to maximize the minimum capacity among all source-destination pairs.

Definition 3.1: (Relay Assignment Problem (RAP)): Given a set ($\mathcal{S}$, $\mathcal{D}$) of source-destination pairs and a set $\mathcal{R}$ of relay nodes, the *Relay Assignment Problem* seeks for a relay assignment σ such that the total capacity $C^{\sigma}_{sum} = \sum_{s_i\in\mathcal{S}} C(s_i, \sigma(s_i))$ is maximized among all the possible relay assignments. □

In the rest of this paper, we use C_{sum} to denote C^{σ}_{sum} when the context is clear.

4. An Optimal Algorithm for RAP

Due to the possibility of sharing a common relay node among multiple source nodes, it is not clear how to formulate RAP as an integer linear programming problem. However, after revealing some special and specific properties of the studied problem, we are able to design a polynomial time optimal algorithm to solve the RAP problem. In this section, we first study these properties, which are essential to the design of our algorithm. Then we present an optimal algorithm for RAP. At the end of this section, we outline some nice properties pertaining to our algorithm.

A. Motivating Example

To illustrate the aforementioned properties, we start with an example consisting of 5 source-destination pairs and 2 relay nodes. Each of the five tables in Table I represents a relay assignment. More specifically, the number in the cell of column s_i and row r_j is the achievable capacity for source node s_i, when relay node r_j is exclusively assigned to it. The ϕ symbol represents direct transmission to the corresponding destination. For example, in Table I(a), the numbers in the first column represent $C_R(s_1, r_1) = 10$, $C_R(s_1, r_2) = 4$ and $C_{DT}(s_1) = 4$. Each highlighted cell represents the current assigned relay node for the corresponding source node. For example, in Table I(b), $\sigma(s_1) = r_1$, $\sigma(s_2) = \sigma(s_4) = \sigma(s_5) = r_2$ and $\sigma(s_3) = \phi$. From Table I(a) to I(d), we illustrate an iteration-by-iteration procedure to improve the total capacity. The total capacity C_{sum} is computed according to (3). In each iteration, we change the relay assignment of the underlined source node from its currently assigned relay node to transmitting directly. For example, in Table I(b), we change the relay assignment of s_3 from r_1 to ϕ and improve the total capacity from 17 to 20. Note that during this procedure, it seems that we can improve the total capacity by changing the assignment of the source node with minimum C_R among all the source nodes sharing the same relay node and letting it transmit directly to its destination. In section 4-B, we will prove that this is not a coincidence but an inherent property.

	s_1	s_2	s_3	s_4	s_5
r_1	10	7	6	6	8
r_2	4	8	4	10	9
ϕ	4	2	1	3	1

(a) $C_{sum} = \frac{10+6}{2} + \frac{8+10+9}{3} = 17$

	s_1	s_2	s_3	s_4	s_5
r_1	10	7	6	6	8
r_2	4	8	4	10	9
ϕ	4	2	1	3	1

(b) $C_{sum} = 10 + \frac{8+10+9}{3} + 1 = 20$

	s_1	s_2	s_3	s_4	s_5
r_1	10	7	6	6	8
r_2	4	8	4	10	9
ϕ	4	2	1	3	1

(c) $C_{sum} = 10 + \frac{10+9}{2} + 2 + 1 = 22.5$

	s_1	s_2	s_3	s_4	s_5
r_1	10	7	6	6	8
r_2	4	8	4	10	9
ϕ	4	2	1	3	1

(d) $C_{sum} = 10 + 10 + 2 + 1 + 1 = 24$

	s_1	s_2	s_3	s_4	s_5
r_1	10	7	6	6	8
r_2	4	8	4	10	9
ϕ	4	2	1	3	1

(e) $C^*_{sum} = 8 + 10 + 4 + 2 + 1 = 25$

TABLE I: An example with 5 source-destination pairs and 2 relays.

B. Algorithm Details

We first prove the property observed from the example in Section 4-A, which is formally stated in Lemma 4.1.

Lemma 4.1: Let σ be a relay assignment, where $n_j > 1$ for some relay node $r_j \in \mathcal{R}$. Let $s_i \in \mathcal{S}(r_j)$ be the source node with the minimum C_R, i.e., $C_R(s_i, r_j) = \min_{s_k \in \mathcal{S}(r_j)} C_R(s_k, r_j)$. If we let s_i directly transmit to the destination d_i, instead of using r_j, while keeping others the same, the total capacity increases. That is $C^{\sigma'}_{sum} > C^{\sigma}_{sum}$, where $\sigma'(s_i) = \phi$ and $\sigma'(s_k) = \sigma(s_i)$ for all $s_k \neq s_i$. □

Proof. Let $\mathcal{S}'(r_j) = \mathcal{S}(r_j) \setminus \{s_i\}$. If s_i directly transmits to d_i instead of employing r_j for CC, we have

$$\begin{aligned}
& C^{\sigma'}_{sum} - C^{\sigma}_{sum} \\
&= C(s_i, \phi) + \sum_{s_k \in \mathcal{S}'(r_j)} \left(\frac{C_R(s_k, r_j)}{n_j - 1} - \frac{C_R(s_k, r_j)}{n_j} \right) - \frac{C_R(s_i, r_j)}{n_j} \\
&= C(s_i, \phi) + \left(\sum_{s_k \in \mathcal{S}'(r_j)} \frac{C_R(s_k, r_j)}{n_j(n_j - 1)} - \frac{C_R(s_i, r_j)}{n_j} \right) \\
&\geq C(s_i, \phi) + \left(C_R(s_i, r_j) \sum_{s_k \in \mathcal{S}'(r_j)} \frac{1}{n_j(n_j - 1)} - \frac{C_R(s_i, r_j)}{n_j} \right) \\
&= C(s_i, \phi) + \left(C_R(s_i, r_j)(n_j - 1) \cdot \frac{1}{n_j(n_j - 1)} - \frac{C_R(s_i, r_j)}{n_j} \right) \\
&= C(s_i, \phi) > 0.
\end{aligned}$$

Therefore, we complete the proof. ■

According to Lemma 4.1, we can always improve the total capacity if there exists a relay node shared by more than one source node in the current relay assignment. Unfortunately, the example in Section 4-A shows that this procedure may lead to a *local optimum*, as shown in Table I(d), whereas an optimal assignment is shown in Table I(e). However, Lemma 4.1 implies a nice property pertaining to the optimal relay assignment for the RAP problem.

Lemma 4.2: In the optimal solution σ^* to RAP, each relay node is assigned to at most one source node, i.e., $n_j \leq 1$ for $\forall r_j \in \mathcal{R}$. □

Proof. Assume to the contrary that there exists a relay node $r_j \in \mathcal{R}$ that is assigned to more than one source nodes, i.e. $n_j > 1$. By Lemma 4.1, we can obtain a new relay assignment with strictly higher total capacity by changing one of the source nodes in $\mathcal{S}(r_j)$ to transmit directly to the destination. This contradicts the optimality of the relay assignment σ^*. Therefore, each relay node is assigned to at most one source node in the optimal solution. ■

Surprisingly, although we allow multiple source nodes to share a common relay node in our model, an optimal relay assignment preferably assigns a relay node to at most one source node to achieve the maximum total capacity. On the other hand, we know that each source node will either employ a relay node for CC or transmit to the destination directly, but not both at the same time. This one-to-one matching relation in the optimal solution indicates that we can transform any instance of RAP into that of the *Maximum Weighted Bipartite Matching* (MWBM) problem [13] and solve it using corresponding algorithms.

For any instance $(\mathcal{S}, \mathcal{D}, \mathcal{R})$ of the RAP problem, we construct an instance $G = (\mathcal{U}, \mathcal{V}, w)$ of the MWBM problem as follows. Let a set $\mathcal{U}$ of vertices represent $\mathcal{S}$. Let a set $\mathcal{V}$ of vertices represent $\mathcal{D} \cup \mathcal{R}$. We set $w(s_i, r_j) = C_R(s_i, r_j)$ for all $s_i \in \mathcal{U}, r_j \in \mathcal{R}$, and set $w(s_i, d_i) = C_{DT}(s_i)$ for all $1 \leq i \leq n$. The transformed MWBM instance of the example in Section 4-A is shown in Fig 3. For clarity purpose, we put no labels on edges.

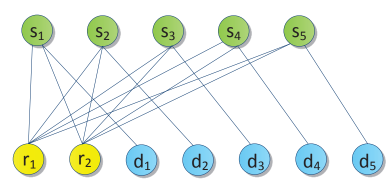


Fig. 3: RAP transformed into a maximum weighted bipartite matching problem.

Now we are ready to present our optimal algorithm for RAP. The detailed pseudo-code is illustrated in Algorithm 1.

Next we give a correctness proof and analyze the computational complexity of Algorithm 1.

Theorem 4.1: Algorithm 1 guarantees to find an optimal relay assignment for RAP in time bounded by $O(n(n + m)\log(n + m) + n^2 m)$.

Proof. We prove the correctness and the running time separately.

Correctness Analysis: First, Lemma 4.2 assures that each relay node is assigned to at most one source node in the optimal relay assignment. In other words, each source node either transmits directly to its destination or use cooperative communication with the help of a relay node that is not shared

Algorithm 1: Optimal Algorithm for RAP

Input : A set $\mathcal{S}$ of source nodes, a set $\mathcal{D}$ of destination nodes, and a set $\mathcal{R}$ of relay nodes.
Output: A relay assignment σ.
1 Construct a set $\mathcal{U}$ of n vertices corresponding to $\mathcal{S}$;
2 Construct a set $\mathcal{V}$ of $n+m$ vertices corresponding to $\mathcal{D} \cup \mathcal{R}$;
3 Construct a set $\mathcal{E}$ of edges, where $(s_i, v) \in \mathcal{E}$ if $v = d_i$ or $v \in \mathcal{R}$;
4 **for** $i = 1$ **to** n **do**
5 | $w(s_i, d_i) \leftarrow C_{DT}(s_i)$;
6 **end**
7 **for** $\forall s_i \in \mathcal{U}$ and $\forall r_j \in \mathcal{R}$ **do**
8 | $w(s_i, r_j) \leftarrow C_R(s_i, r_j)$;
9 **end**
10 Apply an MWBM algorithm to find a maximum weighted matching $\mathcal{M}^*$ in graph $G = (\mathcal{U}, \mathcal{V}, w)$;
11 **for** $(s_i, v) \in \mathcal{M}^*$ **do**
12 | **if** $v \in \mathcal{R}$ **then** $\sigma^*(s_i) = v$;
13 | **else** $\sigma^*(s_i) = \phi$;
14 **end**
15 **return** σ^*.

with any other source nodes. Therefore, each optimal relay assignment can be mapped to a matching in the graph G constructed from Line 1 to Line 9. Assume that there exists another relay assignment σ' resulting in a higher capacity than the one σ^* returned by Algorithm 1. After we map it back to a matching in graph G, we obtain a matching $\mathcal{M}'$ with higher weight than that of $\mathcal{M}^*$ corresponding to σ^*. It contradicts the fact that $\mathcal{M}^*$ is a maximum weighted matching in G. Hence, σ^* is an optimal relay assignment.

Running Time: Note that the most time consuming component in Algorithm 1 is the MWBM algorithm in Line 10. Thus we focus our analysis on this algorithm. The MWBM problem can be solved using a modified shortest path search in augmenting path algorithm. If the Dijkstra algorithm with Fibonacci heap is used, the running time is $O(\min\{|\mathcal{U}|, |\mathcal{V}|\} \cdot ((|\mathcal{U}| + |\mathcal{V}|)\log(|\mathcal{U}| + |\mathcal{V}|) + |\mathcal{E}|))$ [4]. Since $|\mathcal{U}| = n$, $|\mathcal{V}| = n+m$ and $|\mathcal{E}| = nm + n = O(nm)$, the running time of Algorithm 1 is bounded by $O(n(n+m)\log(n+m) + n^2 m)$. ■

Remark 1. Algorithm 1 works regardless of which CC mode is used in the network. Even for a hybrid network where some nodes use AF mode and some use DF mode, our algorithm can still find the optimal assignment.

Remark 2. From the description of Algorithm 1, it is clear that our algorithm is independent of the relation between the number of source-destination pairs and that of relay nodes, i.e., whether $n > m$ or $n \leq m$. Because of the way we construct the bipartite graph, we can always find an assignment for each source node.

Remark 3. Algorithm 1 guarantees to find a relay assignment such that the capacity for each source-destination pair is no less than that under direct transmission. The reason is that if the final capacity of certain source-destination pair is less than that under direct transmission, we can assign it to transmit to the destination directly. It is clear that this change would not reduce the capacities of other source-destination pairs, but can increase the capacity of current source-destination pair. Thus we can obtain a better relay assignment, contradicting the optimality of Algorithm 1.

5. Numerical Results

In this section, we evaluate the performance of our algorithm through extensive experiments. The performance includes both the running time and the total capacity.

A. Experiment Setup

We consider a wireless network, where wireless nodes are randomly distributed in a 1000×1000 square. We follow the same parameter settings as in [12]. Let the bandwidth W be 22 MHz for all channels. The transmission power is the same for each node, i.e., $P_i^s = P_j^r = 1$ Watt for all $s_i \in \mathcal{S}$ and $r_j \in \mathcal{R}$. For the transmission model, we assume that the path loss exponent $\alpha = 4$ and the abient noise $N_0 = 10^{-10}$.

In the experiments, we have two parameters, the number of source-destination pairs n and the number of relay nodes m. We varied both n and m from 50 to 400 with increment of 50. For each setting, we randomly generated 10 instances and averaged the results.

Since this paper is the first work on the problem of relay assignment in cooperative networks with the objective to maximize the total capacity, we compared our algorithm, denoted as *OPT*, with the following algorithms. [1]

Greedy Assignment Algorithm (GAA): We assign a relay node to each source node iteratively. In each iteration, we greedily assign a relay node to the source node or let the source node transmit directly, such that the total capacity of the current assignment is maximized.

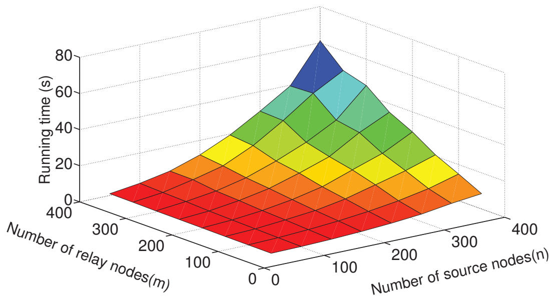
Direct Transmission Algorithm (DTA): We let each source node directly transmit to its destination. That is the total capacity returned by this algorithm is $C_{sum} = \sum_{s_i \in \mathcal{S}} C_{DT}(s_i, d_i)$. DTA serves as a lower bound of the total capacity of the network after the relay assignment.

ORA [12]: The basic idea of ORA is to adjust the assignment iteratively, starting from any arbitrary initial assignment. In each iteration, ORA identifies the source node with currently minimum capacity among all source nodes and searches a better relay node for it. Although ORA is not intentionally designed for the RAP problem, we include it in the comparison for the sake of completeness.

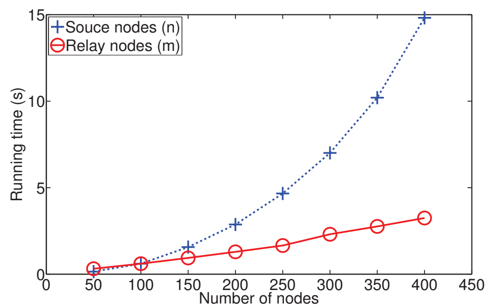
B. Result Analysis

1) Running Time: Fig. 4 shows the running time of our algorithm applied to networks with different size. We observe that our algorithm can solve the RAP problem for a network of 1200 nodes (400 source-destination pairs and 400 relay nodes) within roughly 60 seconds. To have a clear picture about the impacts of the number of source-destination pairs and the number of relay nodes on the running time, we show

[1] We have discussed the difference from [12] in Section 2 and Section 3.



(a) Running time as a function of n and m



(b) Running time as a function of n or m

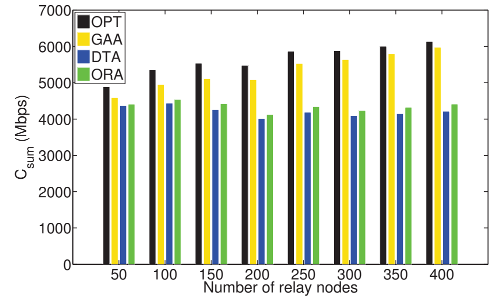
Fig. 4: Scalability

the results in Fig. 4(b). More specifically, to see the impact of the number of source-destination pairs, we varied the value from 50 to 400 while keeping the number of relay nodes fixed at 200, and vice versa. Fig. 4(b) confirms our theoretical analysis on the running time of our algorithm, which proves that the running time is roughly proportional to n^2m in the case n and m are close to each other.

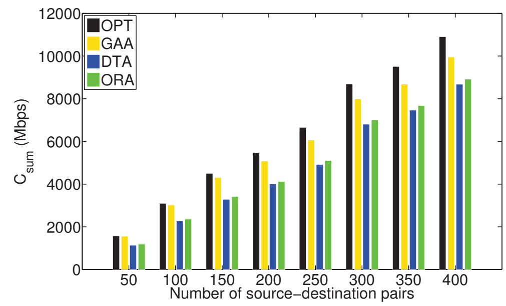
2) Total Capacity: Fig. 5 shows the total capacity of the relay assignments achieved by different algorithms. As expected, our algorithm has the best performance while DTA has the worst. Surprisingly, we observe that GAA has a performance which is only slightly worse than that of our algorithm, especially when the number of relay nodes is greater than the number of source-destination pairs. The reason is that some source nodes may not need to compete with other source nodes for their best relay nodes. Therefore, we may have the same assignment for these source nodes in both OPT and GAA.

6. Conclusions

In this paper, we studied the problem of relay assignment in cooperative networks such that the total capacity is maximized among all the possible assignments. Different from previous work, we allowed the relay node to be shared by multiple source-destination pairs and proposed a corresponding formulation for capacity calculation. Then we developed a polynomial time algorithm to optimally solve this problem. We also showed that our algorithm has three nice properties: (i) it works regardless of the CC mode used in the network; (ii) it is independent of the relation between the number of source-destination pairs and that of relay nodes; and (iii) it guarantees for each source-destination pair to have capacity no less than that under direct transmission.



(a) Varying the number of relay nodes



(b) Varying the number of source-destination pairs

Fig. 5: Results on total capacity

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