

# Truthful Auction for Cooperative Communications

Dejun Yang  
Arizona State University  
Tempe, AZ 85287  
dejun.yang@asu.edu

Xi Fang  
Arizona State University  
Tempe, AZ 85287  
xi.fang@asu.edu

Guoliang Xue  
Arizona State University  
Tempe, AZ 85287  
xue@asu.edu

## ABSTRACT

On one hand, cooperative communication has been gaining more and more popularity since it has great potential to increase the capacity of wireless networks. On the other hand, the applications of cooperative communication technology are rarely seen in reality, even in some scenarios where the demands for bandwidth-hungry applications have pushed the system designers to develop innovative network solutions. A main obstacle lying between the potential capability of channel capacity improvement and the wide adoption of cooperative communication is the lack of incentives for the participating wireless nodes to serve as relay nodes. Hence, in this paper, we design TASC, an auction scheme for the cooperative communications, where wireless node can trade relay services. TASC makes an important contribution of maintaining truthfulness while fulfilling other design objectives. We show analytically that TASC is truthful and has polynomial time complexity. Extensive experiments show that TASC can achieve multiple economic properties without significant performance degradation compared with pure relay assignment algorithms.

## Categories and Subject Descriptors

C.2.1 [Computer-Communication Networks]: Network Architecture and Design

## General Terms

Design, Economics, Algorithm

## Keywords

Cooperative Communication Auctions, Scheme Design

## 1. INTRODUCTION

Cooperative communication [13] has been shown to have great potential to increase the channel capacity between two wireless devices. It essentially exploits the nature of broadcast and the relaying capability of other nodes to achieve spatial diversity. Yet the applications of cooperative communication technology are rarely seen in reality, even in some scenarios where capacity demand continually grows. The cellular network is one such example. The demands for bandwidth-hungry multimedia applications have pushed the system designers to develop innovative network solutions. This is mirrored by the exponentially fast growth of 3G/4G wireless networks. Cell phone carrier companies spend billions of dollars on building the infrastructures. As

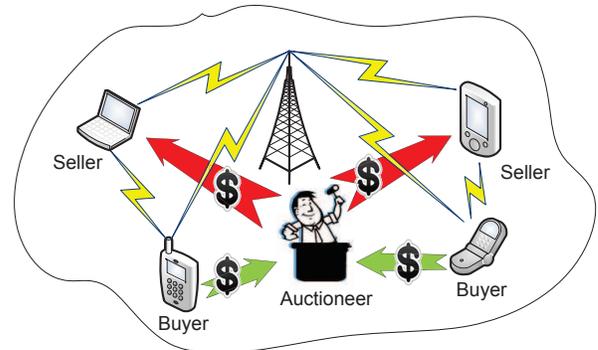


Figure 1: Auction for cooperative communications. Relay nodes (sellers) offer prices to sell their relay services. Source nodes (buyers) bid these services for cooperative communication. The base station (auctioneer) determines winners and clearing prices.

the second largest cell phone carrier company in the U.S., AT&T plans to invest 19 billion dollars on the improvement of 3G networks next year [1]. In contrast to 3G/4G wireless networks, cooperative communication technology does not require extra infrastructure and offers the advantage of flexibility. A main obstacle lying between the potential capability of channel capacity improvement and the wide adoption of cooperative communication is the lack of incentives for the participating wireless nodes to serve as relay nodes. *Why would a cell phone carrier be willing to relay the traffic of another carrier at the cost of its own resource?* One answer to this question is to let the relay node have monetary value in return. Therefore there must be a trade between the wireless node requesting relay service and the one providing such service. Auction is one of the most popular trading form [12], as it allows competitive price discovery and fair and efficient resource allocation.

An auction involving both buyers and sellers is called a *double auction*. To be more specific, the double auction scheme designed in this paper falls into the category of *single-round multi-item double auction*. In this auction scheme, as shown in Figure 1,  $n$  buyers are interested in multiple items from  $m$  sellers. However, each buyer needs at most one item at the end of the auction and each seller can sell its item to at most one buyer. The whole auction procedure processes in a single round fashion. Fairly surprisingly, little work has been done in either networking literature or economics literature. Existing double auction schemes [6,

10, 14, 17, 18] cannot be directly applied to the cooperative communication auction. We will give a brief review on the related work in Section 2.

The auction design is a crucial aspect of trading market, because the auction scheme not only directly defines the trading rules, but also implicitly defines the behaviors of participating agents. Specifically, truthfulness (also called strategy-proofness) is the most critical property of auction scheme. An auction scheme is truthful if revealing the truthful private value is every participating agent’s dominant strategy no matter what strategies other agents are doing. It has been shown both theoretically and practically that an auction could be vulnerable to market manipulation and produce very poor outcomes if this property is not guaranteed [11]. Besides truthfulness, the following properties are also desirable when designing an auction scheme: 1) *Individual Rationality*: each agent participating in the auction can expect a non-negative profit; 2) *Budget Balance*: the auctioneer should finish the auction with no profit loss; 3) *System Efficiency*: the sum of valuations of all agents is optimized, e.g. the total capacity in this paper. Unfortunately, the well-known result from [16] shows that no double auction mechanism can achieve truthfulness, budget-balance, and efficiency at the same time, even putting individual rationality to aside. As our goal of this work is to stimulate the participation of wireless nodes in relay services, we focus our design on satisfying truthfulness, individual rationality and budget balance.

In this paper, we design a Truthful Auction Scheme for Cooperative communications (TASC). The main contributions of this paper are as follows. Firstly, we are the *first* to design a *truthful* auction scheme for cooperative communications, named TASC. TASC implicitly makes it the dominant strategy to bid or ask truthfully for participating agents, thereby eliminating the fear of market manipulation and the overhead of strategizing over others. Secondly, besides being truthful, TASC is also *individually rational* and *budget-balanced*. To the best of our knowledge, this is also the first truthful multi-item double auction scheme even in the economic literature. We hope our study can incite more attention on this type of auction from other researchers. Thirdly, TASC allows the auctioneer to choose any relay assignment algorithm based on its performance requirement. For example, the maximum weighted matching algorithm can be used to maximize the total capacity; Algorithm O-RA [20] can be used to maximize the minimum capacity; and the maximum matching algorithm can be used to maximize the number of successful trades. Last but not least, extensive experiments confirm the truthfulness of TASC, and show that TASC achieves all the required properties with limited degradation on the system efficiency.

The remainder of this paper is organized as follows. First, we briefly review existing related double auction schemes in the economic literature in Section 2. Then we provide an overview of the necessary preliminaries and formulate the problem studied in this paper in Section 3. In Section 4, we discuss the challenges of designing a truthful double auction scheme with required economic properties. Next we give the detailed design of our auction scheme TASC in Section 5. Extensive experiment results are presented in Section 6. Finally we conclude our paper in Section 7.

## 2. RELATED WORK

In this section, we give a brief review on related work. We categorize the related work into two groups, of which one is the related auction schemes from economics literature and the other is specifically for cooperative communications from networking literature.

Although auction theory has been extensively studied in the economics literature, the existing auction designs cannot fully satisfy the required properties stated in Section 1. We summarize the most related works in Table 1. In this table, we list the major differences between the existing works and the auction scheme designed in this paper. Here the heterogeneity of trading items plays an important role in the auction design. It makes the design more challenging as each buyer has preferences on different items from different sellers. Besides the difference listed in the table, some of the existing schemes are also multi-round auctions [6, 2, 15]. Multi-round auction is unsuitable for the cooperative communications, where timeliness is a necessary requirement and large communication overhead is unfavorable.

Existing Work	Heter. Item	Double Auction	Truthful
[6]	✓	✗	✓
[18]	✗	✓	✗
[14]	✗	✓	✓
[3]	✗	✓	✓
[17]	✓	✓	✗
[7]	✓	✓	✗
[10]	✗	✓	✓
[2]	✓	✗	—
[15]	✓	✗	—
This paper	✓	✓	✓

Table 1: Existing auction schemes. “—” means that the corresponding property is unknown.

There are few studies on the auction design for cooperative communications in networking literature, among which the works in [9, 19, 22] are most related to our work. In [19], Shastry and Adve proposed a pricing-based system to stimulate the cooperation via payment to the relay node. In [22], Wang *et al.* employed a buyer/seller Stackelberg game, where a single buyer tries to buy services from multiple relays. The buyer announces its selection of relays and the required transmission power, then the relays ask proper prices to maximize their profits. In [9], Huang *et al.* proposed two auction mechanisms, which are essentially repeated games. In each auction mechanism, each user iteratively updates its bid to maximize its own utility function with the knowledge of others’ previous bids. With a common drawback, none of the above works considered truthfulness, which is critical to the auction scheme.

## 3. PRELIMINARIES AND PROBLEM FORMULATION

### 3.1 Cooperative Communications

We use a well-known three-node example in Figure 2 to describe the essence of cooperative communications (CC). In this example,  $s$  is the source node that transmits information,  $d$  is the destination node that receives information and

$r$  is the relay node that both receives and transmits information to enhance the communication between the source and the destination. CC proceeds in a frame-by-frame fashion. Each frame is divided into two time slots. The source  $s$  transmits data to the destination  $d$  in the first time slot. Due to the broadcast nature, relay node  $r$  can overhear this transmission. In the second time slot,  $r$  forwards the data to  $d$  using different techniques depending on different CC modes. There are two CC modes, *Amplify-and-Forward* (AF) and *Decode-and-Forward* (DF) [13]. For details about AF and DF, we refer interested readers to [13]. We use  $c_R(s, r, d)$  to denote the achievable capacity under CC and  $c_D(s, d)$  to denote the achievable capacity without CC.

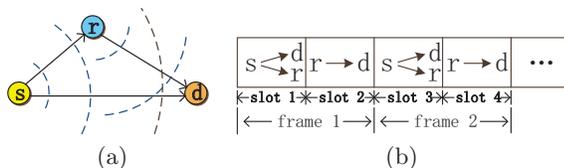


Figure 2: A three-node example for CC

### 3.2 Problem Model

In this paper, we consider a static ad hoc wireless network consisting of  $n$  source-destination pairs  $\{s_1, d_1; s_2, d_2; \dots; s_n, d_n\}$  and a set  $\mathcal{R} = \{r_1, r_2, \dots, r_m\}$  of  $m$  relay nodes. We use  $\mathcal{S} = \{s_1, s_2, \dots, s_n\}$  to denote the set of source nodes and  $\mathcal{D} = \{d_1, d_2, \dots, d_n\}$  to denote the set of destination nodes. We assume that there is a base station acting as a central control and an auctioneer in the auction scheme, e.g. the base station in the cellular networks, where  $d_i$  is the base station for all  $s_i$ 's, as shown in Figure 1.

We design the cooperative communication auction as a *single-round multi-item double auction*. In this auction, source nodes are *buyers*, relay nodes are *sellers*, and the base station is the *auctioneer*. Throughout this paper, we may use source node and buyer, relay node and seller, and base station and auctioneer interchangeably. For narration convenience, we call both buyers and sellers *agents* in general. Buyers bid for relay services for cooperative communication, while sellers offer cooperative services at the cost of resources, e.g. energy, and receive monetary payment in return. For each buyer, it has different valuations of the relay nodes as it can achieve different capacities by cooperating with different relay nodes. Let  $V_i^j$  be buyer  $s_i$ 's true valuation of relay service from seller  $r_j$ , which describes the true price that  $s_i$  is willing to pay for the relay service. Let  $\mathbf{V}_i = (V_i^1, V_i^2, \dots, V_i^m)$  be the true valuation vector of buyer  $s_i$ . Obviously, we have  $V_i^j > 0$  if  $c_R(s_i, r_j, d_i) > c_D(s_i, d_i)$  and  $V_i^j = 0$  otherwise. A buyer has no incentive to buy the relay service which cannot provide higher capacity than transmitting directly. Similarly, let  $C_j$  be seller  $r_j$ 's true cost of providing relay service, which is for example related to the energy consumption. A seller does not differentiate among buyers as it uses the same transmission power. We assume that each buyer wants at most one relay to facilitate the cooperative communication. A recent work by Zhao *et al.* [23] shows that it is sufficient for a source node to choose the best relay node even when multiple are available to achieve full diversity. We also assume that each relay node can be shared by at most one source node as it would provide different capacity from what

the buyer expects otherwise.

The auction is a sealed-bid auction. Following the terminology in auction theory, we refer the price submitted by a buyer and a seller as *bid* and *ask*, respectively. Each buyer (resp. seller) submits its private bid (resp. ask) to the auctioneer and has no knowledge about others. We assume that both asks and bids are static and will not change during the auction. At the beginning of the auction, each buyer  $s_i$  submits a bid vector  $\mathbf{B}_i = (B_i^1, B_i^2, \dots, B_i^m)$ , where  $B_i^j$  is the bid for seller  $r_j$ .  $\mathbf{B}_i$  may or may not be the same as its true valuation vector  $\mathbf{V}_i$ . Each seller  $r_j$  submits its ask  $A_j$ , which may or may not be its true cost  $C_j$ . Let  $\mathbb{B} = (\mathbf{B}_1; \mathbf{B}_2; \dots; \mathbf{B}_n)$  represent the bid matrix consisting of bid vectors submitted by all buyers. Similarly, let  $\mathbf{A} = (A_1, A_2, \dots, A_m)$  represent the set of asks submitted by all sellers. Let  $\mathbf{B}_i^{-j} = (B_i^1, \dots, B_i^{j-1}, B_i^{j+1}, \dots, B_i^m)$  denote the bid vector of buyer  $s_i$  with bid  $B_i^j$  removed. Let  $\mathbb{B}_{-i} = (\mathbf{B}_1; \dots; \mathbf{B}_{i-1}; \mathbf{B}_{i+1}; \dots; \mathbf{B}_n)$  denote the bid matrix with  $s_i$ 's bid vector  $\mathbf{B}_i$  removed. Let  $\mathbb{B}^i$  denote the bid matrix with  $s_i$ 's bid vector changed to  $\mathbf{B}$ . We have  $\mathbf{A}_{-j}$  and  $\mathbf{A}^j$  defined in similar ways. Given  $\mathcal{S}$ ,  $\mathcal{R}$ ,  $\mathcal{D}$ ,  $\mathbb{B}$  and  $\mathbf{A}$ , the auctioneer decides the winners, including both winning buyers and winning sellers, allocates the relay nodes to the source nodes, and determines the *clearing price* for both winning buyers and winning sellers according to the designed auction scheme. Let  $\mathcal{S}_w \subseteq \mathcal{S}$  be the set of winning buyers and  $\mathcal{R}_w \subseteq \mathcal{R}$  be the set of winning sellers. Let  $\sigma : \{i : s_i \in \mathcal{S}_w\} \rightarrow \{j : r_j \in \mathcal{R}_w\}$  be the relay node assignment decided by the auctioneer. Note that  $\sigma(\cdot)$  is actually a one-to-one mapping from the indices of winning buyers to those of winning sellers. Therefore,  $\sigma^{-1}(j)$  is the index of the source node that relay node  $r_j$  is assigned to. Let  $P_i^b$  be the price that the winning buyer  $s_i$  needs to pay. Let  $P_j^s$  be the payment the auctioneer pays the winning seller  $r_j$ . Then the *utility* of buyer  $s_i \in \mathcal{S}$  is defined as

$$U_i^b = \begin{cases} V_i^{\sigma(i)} - P_i^b & \text{if } s_i \in \mathcal{S}_w, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Accordingly, the *utility* of seller  $r_j \in \mathcal{R}$  is defined as

$$U_j^s = \begin{cases} P_j^s - C_j & \text{if } r_j \in \mathcal{R}_w, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

The notations in this paper are summarized in Table 2.

### 3.3 Economic Properties

The design of the auction scheme heavily depends on the desired properties. In the following, we introduce four most common economic properties.

- *Truthfulness*: An auction is truthful if revealing truthful private valuation (resp. cost) is the dominant strategy for each buyer (resp. seller). In other words, no buyer (resp. seller) can improve its utility by submitting a bid (resp. an ask) different from its true valuation (resp. cost), no matter how others submit.
- *Individual Rationality*: An auction is individually rational if no winning buyer will be charged more than its bid and no winning seller will be paid less than its ask, i.e.,  $B_i^{\sigma(i)} \geq P_i^b, A_j \leq P_j^s$  for all  $s_i \in \mathcal{S}_w$  and  $r_j \in \mathcal{R}_w$ . This property ensures that both source nodes and relay nodes have incentives to participate in the cooperative communication.

Symbol	Meaning
$s, r$	source node (buyer), the relay node (seller)
$\mathcal{S}$	set of source nodes (buyers)
$\mathcal{D}$	set of destination nodes
$\mathcal{R}$	set of relay node (sellers)
$\mathcal{S}_w$	winning set of buyers
$\mathcal{R}_w$	winning set of sellers
$\mathcal{S}_c$	candidate winning set of buyers
$\mathcal{R}_c$	candidate winning set of sellers
$\mathbb{S}$	ordered list of buyers
$\mathbb{R}$	ordered list of sellers
$\mathbb{S}_k$	first $k$ buyers in $\mathbb{S}$
$\mathbb{R}_k$	first $k$ sellers in $\mathbb{R}$
$\mathbf{V}_i$	valuation vector of buyer $s_i$
$V_i^j$	valuation of $s_i$ on the relay service from $r_j$
$C_j$	cost of $r_j$
$\mathbb{B}/\mathbf{B}_i/B_i^j$	bid matrix / bid vector of $s_i$ / bid of $s_i$ on $r_j$
$\mathbf{A}/A_j$	ask vector / ask of $r_j$
$x_{-i}$	vector (matrix) with $i$ th element (vector) removed
$x _i y$	vector (matrix) with $i$ th element (vector) replaced by $y$
$\sigma(\cdot)$	mapping function from the indices of buyers to those of sellers
$P_i^b/P_j^s$	price for $s_i$ / payment to $r_j$
$P^b/P^s$	price for all buyers / payment to all sellers
$U_i^b/U_j^s$	utility of $s_i$ / $r_j$
$\tilde{x}$	$x$ with different value
$\Phi(\cdot)$	relay assignment algorithm
$\Psi$	auction scheme for cooperative communications

Table 2: Notations

- *Budget Balance:* An auction is budget-balanced if the total payment by the buyers is no less than the total amount of price paid to the sellers, i.e.,  $\sum_{s_i \in \mathcal{S}_w} P_i^b \geq \sum_{r_j \in \mathcal{R}_w} P_j^s$ .
- *System Efficiency:* An auction is system-efficient if the sum of valuations of all participants is optimized. Put into the context of this paper, system efficiency means the maximization of the total achieved capacity.

### 3.4 Objective

It is desirable to design an auction scheme satisfying all four properties described in the previous section. However, the well-known result in [16] shows that no double auction mechanism can achieve truthfulness, budget-balance, and efficiency at the same time, even putting individual rationality aside. Our ultimate goal is to design an auction scheme that motivates the participation of relay nodes and source nodes in cooperative communications while preventing any agent from rigging its bid or ask to manipulate the market at the same time. Therefore, designing an auction scheme with the first three properties has the highest priority, even at the cost of sacrificing the system efficiency. This methodology was also widely adopted by existing double auctions [3, 7, 10, 24].

In summary, we aim to design a Truthful Auction Scheme for Cooperative communications (TASC), denoted by  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A})$ , where given  $\mathcal{S}$ ,  $\mathcal{R}$ ,  $\mathcal{D}$ ,  $\mathbb{B}$  and  $\mathbf{A}$ , the auctioneer determines the winning buyer set  $\mathcal{S}_w$ , the winning seller set  $\mathcal{R}_w$ , a relay assignment  $\sigma$ , price  $P_i^b$  for each buyer, and payment  $P_j^s$  for each seller, such that:

- For each buyer  $s_i$ ,  $U_i^b$  is maximized when bidding  $\mathbf{V}_i$ ; for each seller  $r_j$ ,  $U_j^s$  is maximized when asking  $C_j$ .

- For each buyer  $s_i$ ,  $B_i^{\sigma(i)} \geq P_i^b$ ; for each seller  $r_j$ ,  $A_j \leq P_j^s$ .
- $\sum_{s_i \in \mathcal{S}_w} P_i^b \geq \sum_{r_j \in \mathcal{R}_w} P_j^s$ .

## 4. CHALLENGES OF COOPERATIVE COMMUNICATION AUCTION DESIGN

In this section, we illustrate the challenges of designing a truthful cooperative communication auction. To better understand these challenges, we show the failures of existing double auction schemes when directly applied to the cooperative communication auction. There are two existing double auction schemes, VCG-based double auction and McAfee double auction. We analyze each of them in Section 4.1 and Section 4.2, respectively.

### 4.1 VCG-based Double Auction

The most well-known auction scheme is the Vickrey-Clarke-Groves (VCG) scheme [4, 8, 21], which can guarantee the truthfulness. In the VCG-based double auction scheme [17], the winners and the assignment between buyers and sellers are determined in a way such that the social welfare  $W = \sum_{s_i \in \mathcal{S}_w} (B_i^{\sigma(i)} - A_{\sigma(i)})$  is maximized. Intuitively, this can be achieved by finding the maximum weighted matching in the bipartite graph  $G = (\mathcal{S}, \mathcal{R}, \mathcal{E}, \delta)$ , where  $(s_i, r_j) \in \mathcal{E}$  if  $B_i^j > 0$  and  $\delta(s_i, r_j) = B_i^j - A_j$  is the weight on edge  $(s_i, r_j)$ . Let  $W^*$  be the optimal value. Let  $W_{-s_i}^*$  be the optimal value when buyer  $s_i$  is removed from the auction. Let  $W_{-r_j}^*$  be the optimal value when seller  $r_j$  is removed from the auction. The price each buyer  $s_i \in \mathcal{S}_w$  needs to pay is

$$P_i^b = B_i^{\sigma(i)} - (W^* - W_{-s_i}^*). \quad (3)$$

The payment each seller  $r_j \in \mathcal{R}_w$  receives is

$$P_j^s = A_j + (W^* - W_{-r_j}^*). \quad (4)$$

Obviously, both  $W^* - W_{-s_i}^*$  and  $W^* - W_{-r_j}^*$  are non-negative for all  $s_i \in \mathcal{S}_w$  and  $r_j \in \mathcal{R}_w$ . Therefore, the VCG-based double auction satisfies the individual rationality property. Furthermore, it has been shown that VCG-based auctions are truthful [17]. The proof closely follows standard Vickrey auction proofs. However, a counter example in Figure 3 shows that the VCG-based double auction scheme is not budget balanced. In this example,  $W^* = 9$ ,  $W_{-s_1}^* = 4$ ,  $W_{-s_2}^* = 7$ ,  $W_{-r_1}^* = 7$ , and  $W_{-r_2}^* = 3$ . Hence  $P_1^b = 10 - (9 - 4) = 5$ ,  $P_2^b = 4 - (9 - 7) = 2$ ,  $P_1^s = 2 + (9 - 7) = 4$  and  $P_2^s = 3 + (9 - 3) = 9$ . The auctioneer finishes the auction with a loss of  $(4 + 9) - (5 + 2) = 6$ .

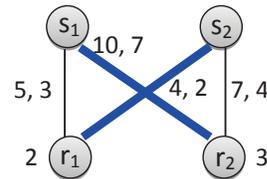


Figure 3: An example showing the budget imbalance of the VCG-based double auction. The two numbers associated with link  $(s_i, r_j)$  are bid  $B_i^j$  and  $\delta(s_i, r_j)$ . The thick blue lines represent the maximum weighted matching. The number besides each seller is its ask.

## 4.2 McAfee Double Auction

In the McAfee double auction [14], items for auction are homogeneous. Buyers have no preference on these items. Therefore each buyer  $s_i$  only submits one bid  $B_i$  and each seller  $r_j$  offers one ask  $A_j$ . The auctioneer starts by sorting the bids in non-increasing order and the asks in non-decreasing order:  $B_{i_1} \geq B_{i_2} \geq \dots B_{i_n}$  and  $A_{j_1} \leq A_{j_2} \leq \dots A_{j_m}$ . The auctioneer then finds the largest  $k$  such that  $B_{i_k} \geq A_{j_k}$  and  $B_{i_{k+1}} < A_{j_{k+1}}$ . Let  $t = \frac{B_{i_{k+1}} + A_{j_{k+1}}}{2}$ . The clearing prices are determined as follows:

$$\begin{cases} P^b = P^s = t & \text{if } A_{j_k} \leq t \leq B_{i_k}, \\ P^b = B_{i_k}, P^s = A_{j_k} & \text{otherwise,} \end{cases}$$

where  $P^b$  is the price charged to each winning buyer and  $P^s$  is the payment that each winning seller receives. Although McAfee double auction satisfies all three properties desired in this paper [14], the homogeneity of auction items makes it unsuitable for the cooperative communication auction without further development.

## 5. OUR AUCTION SCHEME

In this section, we present TASC, a truthful and computationally efficient auction scheme for cooperative communications. We start by giving a brief overview of the design rationale. We then describe the detailed design consisting of two main stages. We use an illustrative example to facilitate the understanding of TASC. Next we show that TASC satisfies the three properties listed at the end of Section 3.4. Finally, we prove that TASC has a polynomial time complexity of  $O(\mathcal{T} + l^2)$ , where  $\mathcal{T}$  is the time complexity of the relay assignment algorithm and  $l = \min\{n, m\}$ .

### 5.1 Overview

Although the VCG-based double auction is superficially closest to the auction we aim to design, the imbalance of the budget is unacceptable to the auctioneer. In contrast, TASC is inspired by the design of McAfee double auction. TASC consists of two stages: *Assignment* and *Winner-Determination & Pricing*. To overcome the limitation of McAfee double auction, we apply an assignment algorithm to find a relay assignment in the assignment stage. In the second stage, we apply McAfee double auction to determine the clearing price for both sellers and buyers. The auctioneer charges all winning buyers the same price and pays all winning sellers the same payment.

### 5.2 Design

We now describe the design of TASC in detail. In the assignment stage, we need to design a new relay assignment algorithm or apply the existing relay assignment algorithms with the requirement of being independent of the buyers' bids and the sellers' asks. The relay assignment algorithm's dependency on either the bids or the asks could make the auction vulnerable to market manipulation. Our procedure for the assignment stage is shown in Algorithm 1.

Depending on the specific scenario, the auctioneer can choose different assignment algorithms ( $\Phi(\cdot)$ ) for different purposes. For example, to maximize the total capacity, the maximum weighted matching algorithm can be applied; to maximize the minimum capacity among all source nodes, Algorithm ORA in [20] can be applied; to maximize the

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#### Algorithm 1: TASC-Asgmnt( $\mathcal{S}, \mathcal{R}, \mathcal{D}$ )

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1 Construct a set  $\mathcal{U}$  of vertices corresponding to  $\mathcal{S}$ ;
2 Construct a set  $\mathcal{V}$  of vertices corresponding to  $\mathcal{R}$ ;
3  $\mathcal{E} \leftarrow \emptyset$ ;
4 forall the  $s_i \in \mathcal{S}, r_j \in \mathcal{R}$  do
5   | if  $c_R(s_i, r_j, d_i) > c_D(s_i, d_i)$  then
6   |   |  $\mathcal{E} \leftarrow \mathcal{E} \cup \{(s_i, r_j)\}$ ;
7   | end
8 end
9  $(\mathcal{S}_c, \mathcal{R}_c, \sigma) \leftarrow \Phi(\mathcal{U}, \mathcal{V}, \mathcal{E}, c_R)$ ;
10 return  $(\mathcal{S}_c, \mathcal{R}_c, \sigma)$ ;

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number of trades, the maximum matching algorithm fulfills the mission. The return values include the candidate winning buyers  $\mathcal{S}_c$ , the candidate winning sellers  $\mathcal{R}_c$  and the assignment  $\sigma$ .

In the winner-determination & pricing stage, we tightly integrate the winner determination and the pricing operation. With the assignment obtained in the previous stage, we can apply McAfee double auction to determine the winners and the clearing prices. The detailed algorithm is shown in Algorithm 2.

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#### Algorithm 2: TASC-WD&Pricing( $\mathcal{S}_c, \mathcal{R}_c, \sigma, \mathbb{B}, \mathbf{A}$ )

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1  $\mathcal{S}_w \leftarrow \emptyset, \mathcal{R}_w \leftarrow \emptyset$ ;
2 Sort all the buyers in  $\mathcal{S}_c$  to get an ordered list
   $\mathbb{S} = \langle s_{i_1}, s_{i_2}, \dots \rangle$  such that  $B_{i_1}^{\sigma(i_1)} \geq B_{i_2}^{\sigma(i_2)} \dots$ ;
3 Sort all the sellers in  $\mathcal{R}_c$  to get an ordered list
   $\mathbb{R} = \langle r_{j_1}, r_{j_2}, \dots \rangle$  such that  $A_{j_1} \leq A_{j_2} \dots$ ;
4 Find the largest  $k$ , such that  $B_{i_k}^{\sigma(i_k)} \geq A_{j_k}$ ;
5 if  $k < 2$  then return  $(\mathcal{S}_w, \mathcal{R}_w, 0, 0)$ ;
6  $(x, y) \leftarrow (i_k, j_k)$ ;
7 // Determine the price and the payment
8  $P^b \leftarrow B_x^{\sigma(x)}, P^s \leftarrow A_y$ ;
9 // Sacrifice one buyer and one seller to ensure
  the truthfulness
10  $\mathcal{S}_w \leftarrow \mathbb{S}_x \setminus \{s_x\}, \mathcal{R}_w \leftarrow \mathbb{R}_y \setminus \{r_y\}$ ;
11 // Determine the final winners
12 for  $s_i \in \mathcal{S}_w$  do
13   | if  $r_{\sigma(i)} \notin \mathcal{R}_w$  then  $\mathcal{S}_w \leftarrow \mathcal{S}_w \setminus \{s_i\}$ ;
14 end
15 for  $r_j \in \mathcal{R}_w$  do
16   | if  $s_{\sigma^{-1}(j)} \notin \mathcal{S}_w$  then  $\mathcal{R}_w \leftarrow \mathcal{R}_w \setminus \{r_j\}$ ;
17 end
18 return  $(\mathcal{S}_w, \mathcal{R}_w, P^b, P^s)$ ;

```

---

For ease of illustration, we introduce more notations and concepts.

- $\mathbb{S}$  denotes an ordered list of buyers sorted in non-increasing order according to their bids on the assigned relay nodes.
- $\mathbb{R}$  denotes an ordered list of sellers sorted in non-decreasing order according to their asks.
- $\mathbb{S}_k$  denotes the sublist of the first  $k$  buyers in  $\mathbb{S}$ .
- $\mathbb{R}_k$  denotes the sublist of the first  $k$  sellers in  $\mathbb{R}$ .
- $\Sigma(\mathbb{S}, \mathbb{R})$  denotes the set of matchings induced by  $\mathbb{S}$  and  $\mathbb{R}$ , i.e.,  $\Sigma(\mathbb{S}, \mathbb{R}) = \{(s_i, r_j) : s_i \in \mathbb{S}, r_j \in \mathbb{R}, j = \sigma(i)\}$ .

- We call the buyer-seller pair, according to which the auctioneer determines the winners and clearing prices, the *boundary pair*. In McAfee double auction (Section 4),  $s_{i_k} - r_{j_k}$  is such a pair.
- Since the winning buyer and the winning seller are pairwise determined, we call  $(s_i, r_{\sigma(i)})$  or  $(s_{\sigma^{-1}(j)}, r_j)$  the *winning pair*.

The main algorithm of TASC is illustrated in Algorithm 3.

---

**Algorithm 3:** TASC( $\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A}$ )

---

- 1  $(\mathcal{S}_c, \mathcal{R}_c, \sigma) \leftarrow \text{TASC-Asgmt}(\mathcal{S}, \mathcal{R}, \mathcal{D});$
  - 2  $(\mathcal{S}_w, \mathcal{R}_w, P^b, P^s) \leftarrow \text{TASC-WD\&Pricing}(\mathcal{S}_c, \mathcal{R}_c, \sigma, \mathbb{B}, \mathbf{A});$
  - 3 **return**  $(\mathcal{S}_w, \mathcal{R}_w, \sigma, P^b, P^s);$
- 

### 5.3 An Illustrative Example

We use a simple example to illustrate the idea of TASC. The achievable capacity matrix, which is also assumed to be the bid matrix, is shown in Table 3(a) and the ask vector is shown in Table 3(b).

	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$	$r_7$
$s_1$	10	4	4	0	0	0	0
$s_2$	0	0	7	3	4	0	8
$s_3$	7	0	0	4	6	0	0
$s_4$	0	6	0	10	4	6	0
$s_5$	0	0	8	0	0	9	4

(a) The capacity matrix (also the bid matrix) and the assignment

seller	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$	$r_7$
ask	3	2	5	6	4	1	7

(b) Asks of sellers

Table 3: An example with 5 source-destination pairs and 7 relay nodes.

In the assignment stage, we assume the auctioneer applies the maximum weighted matching algorithm. The resulting assignment is highlighted in Table 3(a), where, for example, seller  $r_1$  is assigned to buyer  $s_1$ . We draw the assignment as a bipartite graph in Figure 4, where nodes are sorted according to their bids or asks.

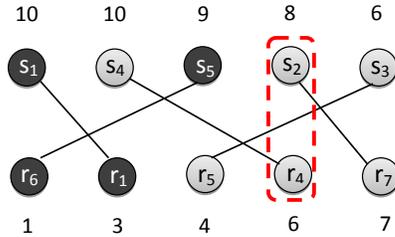


Figure 4: A bipartite graph showing the winner determination and pricing stage of TASC.  $\mathbb{S} = \langle s_1, s_4, s_5, s_2, s_3 \rangle$ ,  $\mathbb{R} = \langle r_6, r_1, r_5, r_4, r_7 \rangle$ .

In the winner-determination & pricing stage, the auctioneer finds that  $k = 4$ . Hence the winning pairs include  $(s_1, r_1)$ , and  $(s_5, r_6)$ . The price each winning buyer needs to pay is  $P^b = 8$  and the payment each winning seller receives is  $P^s = 6$ . The profit the auctioneer makes is  $2 \times (8 - 6) = 4$ .

### 5.4 Proofs of Economic Properties

Having given the detailed design of TASC, we now prove the desired economic properties mentioned in Section 3.4.

**Theorem 1.** TASC is individually rational.  $\square$

**Proof.** For each winning buyer  $s_i \in \mathcal{S}_w \subseteq \mathbb{S}_x$ , we know that  $B_i^{\sigma(i)} \geq B_x^{\sigma(x)} = P^b$ . The same claim also holds for each seller. This completes our proof.  $\blacksquare$

**Theorem 2.** TASC is budget-balanced.  $\square$

**Proof.** Note that we have  $|\mathcal{S}_w| = |\mathcal{R}_w|$  based on the assignment assumption. For each winning buyer  $s_i \in \mathcal{S}_w$  and its assigned winning seller  $r_{\sigma(i)} \in \mathcal{R}_w$ , we have  $P_i^b = P^b = B_x^{\sigma(x)} \geq A_y = P^s = P_j^s$ . Therefore we have  $\sum_{s_i \in \mathcal{S}_w} P_i^b - \sum_{r_j \in \mathcal{R}_w} P_j^s = |\mathcal{S}_w|(P^b - P^s) \geq 0$ , which completes the proof.  $\blacksquare$

**Theorem 3.** TASC is truthful.  $\square$

Before proving Theorem 3, we need to prove a series of lemmas. We show that the auction result of each buyer is partially independent of its bid in Lemma 1, the winner-determination is bid-monotonic [3] (resp. ask-monotonic) for the buyer in Lemma 2 (resp. the seller in Lemma 3), the pricing is bid-independent for buyers in Lemma 4 (resp. ask-independent for the sellers in Lemma 5) and TASC is truthful for buyers in Lemma 6 (resp. sellers in Lemma 7).

Hereafter, we use tilde to differentiate notations with the same meaning, e.g.,  $\tilde{\mathbf{B}}_i$  and  $\mathbf{B}_i$  are two different bid vectors of  $s_i$ . In addition, we define several comparison operators:  $>_j, =_j, <_j$ . We say  $\tilde{\mathbf{B}}_i >_j \mathbf{B}_i$  if  $\tilde{B}_i^j > B_i^j$ ,  $\tilde{\mathbf{B}}_i =_j \mathbf{B}_i$  if  $\tilde{B}_i^j = B_i^j$  and  $\tilde{\mathbf{B}}_i <_j \mathbf{B}_i$  if  $\tilde{B}_i^j < B_i^j$ .

**Lemma 1.** If buyer  $s_i$  is assigned relay  $r_{\sigma(i)}$  in the assignment stage, then the auction result for  $s_i$  is independent of its bids  $\mathbf{B}_i^{-\sigma(i)}$ . In other words, the results of  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \mathbf{B}_i, \mathbf{A})$  and  $\tilde{\Psi} = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \tilde{\mathbf{B}}_i, \mathbf{A})$  are the same, if  $\mathbf{B}_i =_{\sigma(i)} \tilde{\mathbf{B}}_i$ .  $\square$

**Proof.** The assignment stage is independent of bids and asks. In the winner-determination and pricing stage (Algorithm 2), it is clear that both the winner determination and the price charged to the buyer are only dependent on the bid the buyer bids on  $r_{\sigma(i)}$  and the asks  $\mathbf{A}$  of sellers. Therefore, our lemma holds.  $\blacksquare$

Due to the space limitation, we prove the properties for buyers in the following lemmas and only prove one for sellers. Other properties for sellers can be proved in similar ways as we prove for buyers.

**Lemma 2.** If  $s_i$  wins  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \mathbf{B}_i, \mathbf{A})$  by bidding  $\mathbf{B}_i$ , it can also win  $\tilde{\Psi} = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \tilde{\mathbf{B}}_i, \mathbf{A})$  by bidding  $\tilde{\mathbf{B}}_i >_{\sigma(i)} \mathbf{B}_i$ .  $\square$

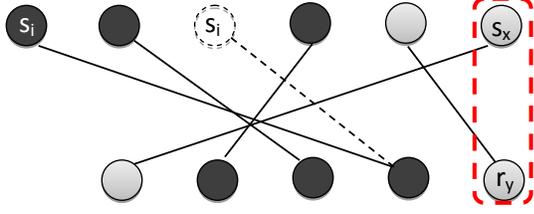


Figure 5: An illustration for Lemma 2

**Proof.** In the assignment stage, since the relay assignment algorithm is independent of bids and asks, if  $s_i$  is assigned a relay node  $r_{\sigma(i)}$  in  $\Psi$ , it is assigned the same relay node in  $\tilde{\Psi}$  as well. Let  $p_i$  and  $\tilde{p}_i$  be  $s_i$ 's positions in  $\mathbb{S}$  and  $\tilde{\mathbb{S}}$ , respectively. An illustration is shown in Figure 5. Because  $\tilde{B}_i^{\sigma(i)} > B_i^{\sigma(i)}$ , the orders in  $\mathbb{S}$  and  $\tilde{\mathbb{S}}$  after  $p_i$  are exactly the same. In addition, we know that the values of  $k$  (Line 4) are the same in both  $\mathbb{S}$  and  $\tilde{\mathbb{S}}$ . Therefore, during the winner determination stage,  $s_x$  and  $r_y$  are still selected as the boundary pair in  $\tilde{\Psi}$ , which implies that buyer  $s_i$  is also a winner in  $\tilde{\Psi}$ . ■

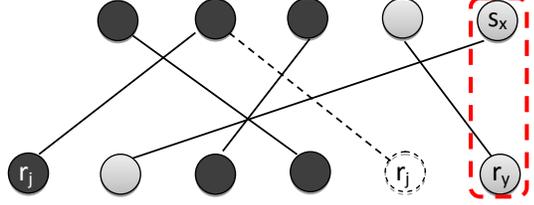


Figure 6: An illustration for Lemma 3

**Lemma 3.** If  $r_j$  wins  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A}^j A_j)$  by asking  $A_j$ , it can also win  $\tilde{\Psi} = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A}^j \tilde{A}_j)$  by asking  $\tilde{A}_j < A_j$ . □

**Proof.** Since the relay assignment algorithm is independent of bids and asks,  $r_j$  is assigned to the same buyer  $s_{\sigma^{-1}(j)}$  in both  $\Psi$  and  $\tilde{\Psi}$ . Let  $q_i$  and  $\tilde{q}_i$  be  $r_j$ 's positions in  $\mathbb{R}$  and  $\tilde{\mathbb{R}}$ , respectively. An illustration is shown in Figure 6. Because  $\tilde{A}_j < A_j$ , the orders in  $\mathbb{R}$  and  $\tilde{\mathbb{R}}$  after  $q_i$  are exactly the same. In addition, we know that the values of  $k$  (Line 4) are the same in both  $\mathbb{R}$  and  $\tilde{\mathbb{R}}$ . Therefore, during the winner determination stage,  $s_x$  and  $r_y$  are still selected as the boundary pair in  $\tilde{\Psi}$ , which implies that seller  $r_i$  is also a winner in  $\tilde{\Psi}$ . ■

**Lemma 4.** If  $s_i$  wins both  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \mathbf{B}_i, \mathbf{A})$  and  $\tilde{\Psi} = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}^i \tilde{\mathbf{B}}_i, \mathbf{A})$  by bidding  $\mathbf{B}_i$  and  $\tilde{\mathbf{B}}_i$ , it is charged the same price, i.e.,  $P^b = \tilde{P}^b$ . □

**Proof.** By Lemma 1, we know that buyer  $s_i$ 's bid  $\mathbf{B}_i^{-\sigma(i)}$  will not change the auction result nor the charged price if it wins the auction. Hence, without loss of generality, we assume that  $\tilde{\mathbf{B}}_i >_{\sigma(i)} \mathbf{B}_i$ . As mentioned in the proof of Lemma 2,  $s_x$  and  $r_y$  are selected as the boundary pair in both  $\Psi$  and  $\tilde{\Psi}$ . According to the pricing strategy, we know that, buyer  $s_i$  is charged the same price,  $P^b = \tilde{P}^b = B_x^{\sigma(x)}$ , in both  $\Psi$  and  $\tilde{\Psi}$ . ■

**Lemma 5.** If  $r_j$  wins both  $\Psi = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A}^j A_j)$  and  $\tilde{\Psi} = (\mathcal{S}, \mathcal{R}, \mathcal{D}, \mathbb{B}, \mathbf{A}^j \tilde{A}_j)$  by asking  $A_j$  and  $\tilde{A}_j$ , it is paid the same payment, i.e.,  $P^s = \tilde{P}^s$ . □

Case	Result	
$\mathbf{B}_i =_{\sigma(i)} \mathbf{V}_i$	$\tilde{U}_i^b = U_i^b$	
$\mathbf{B}_i >_{\sigma(i)} \mathbf{V}_i$	$\mathbf{B}_i: w, \mathbf{V}_i: w$	$\tilde{U}_i^b = U_i^b$
	$\mathbf{B}_i: w, \mathbf{V}_i: l$	$\tilde{U}_i^b \leq U_i^b$
	$\mathbf{B}_i: l, \mathbf{V}_i: l$	$\tilde{U}_i^b = U_i^b$
$\mathbf{B}_i <_{\sigma(i)} \mathbf{V}_i$	$\mathbf{B}_i: w, \mathbf{V}_i: w$	$\tilde{U}_i^b = U_i^b$
	$\mathbf{B}_i: l, \mathbf{V}_i: w$	$\tilde{U}_i^b \leq U_i^b$
	$\mathbf{B}_i: l, \mathbf{V}_i: l$	$\tilde{U}_i^b = U_i^b$

Table 4: Proof logic of Lemma 6.  $w$  means it wins and  $l$  means it loses.

**Lemma 6.** TASC is truthful for buyers. □

**Proof.** We prove this theorem by showing that no buyer  $s_i$  can improve its utility by bidding  $\mathbf{B}_i \neq \mathbf{V}_i$ , i.e.  $\tilde{U}_i^b \leq U_i^b$  for any  $\mathbf{B}_i \neq \mathbf{V}_i$ , where  $\tilde{U}_i^b$  and  $U_i^b$  are the utilities of  $s_i$  when bidding  $\mathbf{B}_i$  and  $\mathbf{V}_i$ , respectively. We examine all the possible cases one by one as shown in Table 4.

• **Case 1:**  $\mathbf{B}_i =_{\sigma(i)} \mathbf{V}_i$

By Lemma 1, we know that buyer  $s_i$  is charged the same price  $P^b$  by bidding  $\mathbf{B}_i$  and  $\mathbf{V}_i$  if  $\mathbf{B}_i =_{\sigma(i)} \mathbf{V}_i$ . Therefore, we have  $\tilde{U}_i^b = V_i^{\sigma(i)} - P^b = U_i^b$ .

• **Case 2:**  $\mathbf{B}_i >_{\sigma(i)} \mathbf{V}_i$

By Lemma 2, we know that it is impossible that buyer  $s_i$  wins the auction by bidding  $\mathbf{V}_i$  but loses by bidding  $\mathbf{B}_i$ . Hence there are three subcases: 1)  $s_i$  wins by bidding both  $\mathbf{V}_i$  and  $\mathbf{B}_i$ ; 2)  $s_i$  wins by bidding  $\mathbf{B}_i$  but loses by bidding  $\mathbf{V}_i$ ; and 3)  $s_i$  loses by bidding both  $\mathbf{V}_i$  and  $\mathbf{B}_i$ . For subcase 1), buyer  $s_i$  is charged the same price  $P^b$  according to Lemma 4. Hence, we have  $\tilde{U}_i^b = U_i^b = V_i^{\sigma(i)} - P^b$ . For subcase 2), we have  $\tilde{U}_i^b = U_i^b = 0$  because  $s_i$  loses in both auctions. Now we focus on subcase 2). Since  $s_i$  wins by bidding  $\mathbf{B}_i$  and loses by bidding  $\mathbf{V}_i$ , we know that  $\tilde{P}^b = B_x^{\sigma(\tilde{x})} \geq V_i^{\sigma(i)}$ , where  $\tilde{x}$  is the index of buyer selected in Line 6 in  $\tilde{\Psi}$ . Hence, we have  $\tilde{U}_i^b = V_i^{\sigma(i)} - \tilde{P}^b \leq 0 = U_i^b$ .

• **Case 3:**  $\mathbf{B}_i <_{\sigma(i)} \mathbf{V}_i$

By Lemma 2, we know that it is impossible that the buyer wins the auction by bidding  $\mathbf{B}_i$  but loses by bidding  $\mathbf{V}_i$ . Hence there are three subcases: 1)  $s_i$  wins by bidding both  $\mathbf{V}_i$  and  $\mathbf{B}_i$ ; 2)  $s_i$  loses by bidding  $\mathbf{B}_i$  but wins by bidding  $\mathbf{V}_i$ ; and 3)  $s_i$  loses by bidding both  $\mathbf{V}_i$  and  $\mathbf{B}_i$ . For subcases 1) and 3), we can prove that  $\tilde{U}_i^b = U_i^b$  following the same analysis as in Case 2. For subcase 2), it is clear that  $\tilde{U}_i^b = 0$  and  $U_i^b \geq 0$ .

We have proved that a buyer cannot improve its utility by submitting a bid vector other than its true valuation vector. This completes the proof. ■

**Lemma 7.** TASC is truthful for sellers. □

**Proof of Theorem 3:** Lemma 6 and Lemma 7 together prove that TASC is truthful. ■

## 5.5 Time Complexity

**Theorem 4.** *The time complexity of TASC is  $O(\mathcal{T} + l^2)$ , where  $\mathcal{T}$  is the time complexity of relay assignment algorithm and  $l = \min\{n, m\}$ .  $\square$*

**Proof.** For the assignment stage, the time complexity depends on the relay assignment algorithm used. For example, the maximum weighted matching algorithm has time complexity of  $O((n+m)^2 \log(n+m) + (n+m)nm)$  [5], Algorithm ORA [20] takes  $O(nm^2)$  time, and the maximum matching algorithm has time complexity of  $O(\sqrt{n+m} \cdot nm)$ . We denote the time complexity of the relay assignment algorithm by  $\mathcal{T}$  in general. In the winner determination & pricing stage, since the input is the assignment result, we have the number of buyers equal to the number of sellers. Obviously, this number, denoted as  $l$ , is not greater than  $\min\{n, m\}$ . Sorting both sellers and buyers takes  $O(l \log l)$  time (Lines 2 and 3). Finding the boundary pair takes  $O(l)$  time (Line 4). Determining the final winning pairs takes  $O(l^2)$  time (Lines 10 to 17). The time complexity of this stage is thus  $O(l^2)$ . Therefore the overall time complexity of TASC is  $O(\mathcal{T} + l^2)$ .  $\blacksquare$

## 6. NUMERICAL RESULTS

In this section, we present extensive experiments to evaluate the performance of TASC and study the economic impact on the system efficiency.

### 6.1 Experiment Setup

We considered a wireless network where nodes are randomly distributed in a  $1000 \times 1000$  square. We followed the same parameter settings as in [20]. Let the bandwidth be 22 MHz for all channels. The transmission power is 1 Watt for all wireless nodes. For the transmission model, we assume that the path loss exponent is 4 and the noise is  $10^{-10}$ . For cooperative communication, DF mode was used. We fixed the number of buyers ( $n$ ) at 100 and varied the number of sellers ( $m$ ) from 50 to 150 with increment of 10. For each setting, we randomly generated 1000 instances and averaged the results. All the tests were run on a Linux PC with 2.00 GHz Intel Pentium CPU and 1.5 GB memory.

For the auction, we assume the buyers' bids are randomly distributed over  $(0, V_{max}]$ , where  $V_{max}$  is set to 4 in most of experiments and varied in the experiments showing the impact of bid distribution on the system efficiency. Similarly we assume the sellers' asks are randomly distributed over  $(0, 1]$ .

The performance metrics in the experiments include agents' utilities, auctioneer's profit, total capacity, number of successful trades and the minimum capacity among all buyers. Throughout all the experiments, we denote the maximum weighted matching algorithm by MWM and the maximum matching algorithm by MM.

### 6.2 Truthfulness of TASC

To verify the truthfulness of TASC, we randomly pick one buyer and one seller, and examine how their utilities change when they bid or ask different values. The results are shown in Figure 7(a)-(c) for the buyer and in Figure 7(d)-(f) for the seller. We note that for each combination of true valuation and the assignment algorithm, no buyer (resp. seller) can improve its utility by bidding (resp. asking) untruthfully.

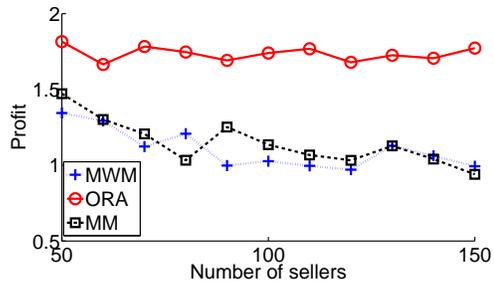


Figure 8: Profit of the auctioneer

### 6.3 Impact on Profit

Although making profit is not the goal of designing TASC, it is still necessary to study the impacts of different assignment algorithms on the profit. Figure 8 plots the profits of the auctioneer when different relay assignment algorithms are applied. The first observation is that the profits are low for all three different relay assignment algorithms, with the maximum profit less than 2. Due to the way we determine the price and the payment, we have  $P^b = P^s$  for many instances. Therefore, the profits are 0 in these instances. Another observation is that the profit decreases with the increase of the number of sellers. This is because as more and more sellers are involved in the auction, the probability that  $P^b = P^s$  is becoming higher.

### 6.4 Impact on System Efficiency

Depending on the system performance requirement, the system efficiency could be the total capacity, the number of successful trades, and the minimum capacity among all the participating buyers. Clearly, since the auctioneer cannot allow all the participating agents to be winners, it is inevitable to have degradation over the pure relay assignment, except for the auction with ORA. When the minimum capacity is the system efficiency, TASC will not degrade the performance since the winner decision is made upon the optimal results. To capture the economic impact on the system efficiency, we plot the degradation of TASC over pure relay assignment algorithms in Figure 9(a). Surprisingly, the degradation is independent of the number of sellers for both MWM and MM.

Next we study the impact of the bid distribution on the system efficiency. Figure 9(b) illustrates the degradation of TASC with both MWM and MM for different values of  $V_{max}$ . We observe that when the maximum bid value  $V_{max}$  increases, the degradation of TASC over the pure relay assignment algorithms decreases. In other words, when buyers have higher true valuations on the services from relay nodes, TASC can achieve all the required economic properties without degrading the system efficiency significantly.

### 6.5 Running Time

To confirm our time complexity analysis in Section 5.5, we illustrate the running time of TASC with different assignment algorithms in Figure 10. We note that the running time increases with the increase of the number of sellers for both MWM and ORA. However, for MM, the running time increases first and then becomes stable after  $m = n$ . This is because the maximum number of matching is limited by  $n$  even when  $m$  keeps increasing.

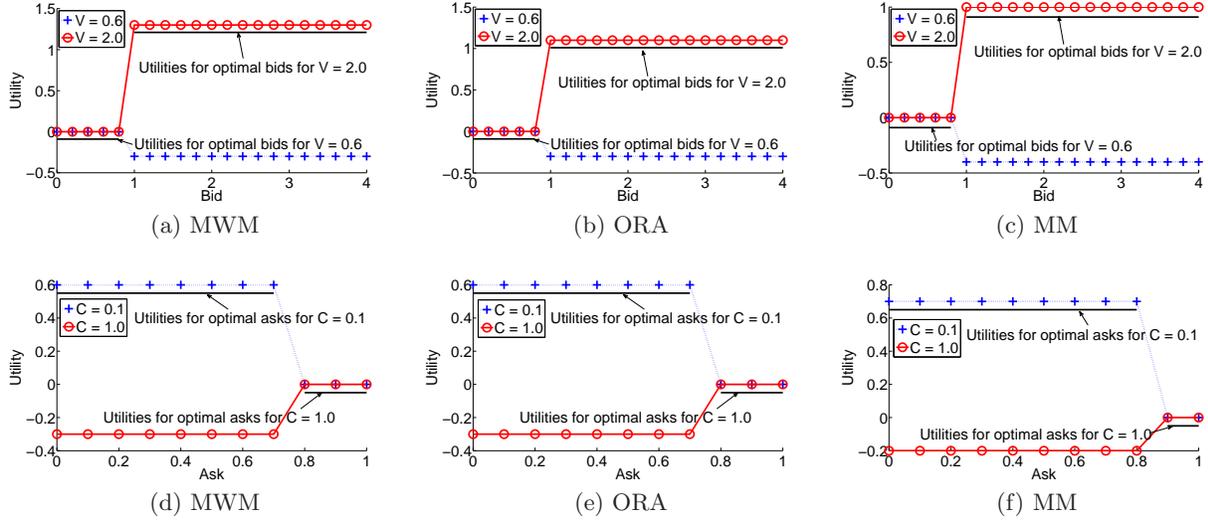


Figure 7: The utilities of a buyer ((a)-(c)) and a seller ((d)-(f)) in auctions with different assignment algorithms, where  $n = m = 100$ . In each auction,  $V$  is the true valuation of the buyer and  $C$  is the true cost of the seller. Two different values are tested for both buyer's true valuation  $V$  and seller's true cost  $C$ . For each different true valuation (resp. cost), the buyer (resp. the seller) cannot improve its utility by submitting bid (resp. offering ask) different from its true valuation (resp. cost).

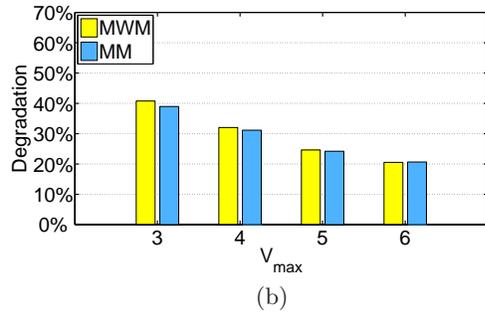
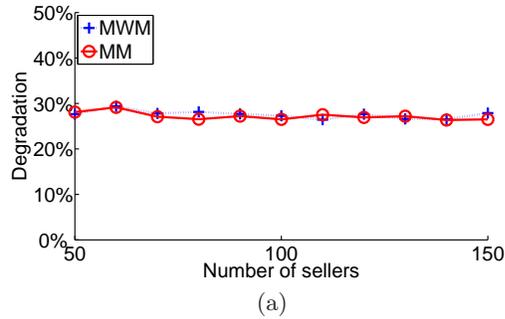


Figure 9: System degradation of TASC over pure relay assignment algorithms

## 7. CONCLUSIONS

In this paper, we have designed TASC, a truthful auction scheme for cooperative communications. To stimulate the participation of wireless devices in relaying traffic for others, TASC allows potential relay nodes to offer prices on their relay services and requires interested source nodes to bid on them. With a careful design, TASC explicitly enforce both

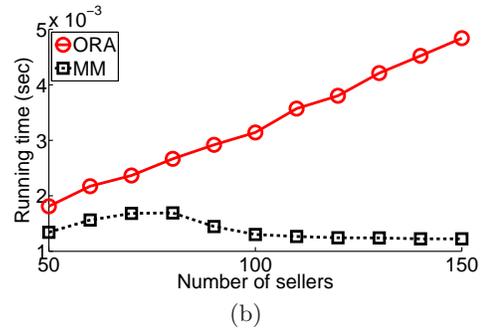
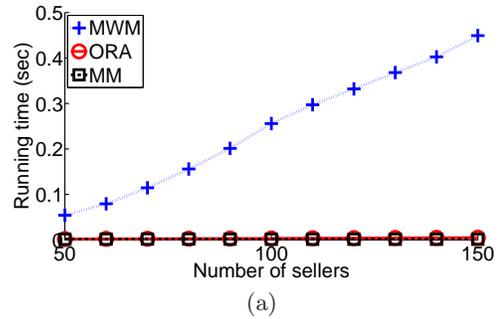


Figure 10: The running time of TASC, where  $n = 100$  and  $m$  is varying from 50 to 150. (a) and (b) use the same set of results, while (b) shows the results without MWM for clarity.

sellers and buyers to submit their true valuations, thereby eliminating the fear of market manipulation and the overhead of strategizing over others for them. Meanwhile, TASC also satisfies individual rationality and budget balance properties. In addition, TASC can use any relay assignment algo-

rithm to achieve different system performance requirements. Extensive experiment results confirm our theoretic analysis of TASC and show that TASC can achieve all the required economic properties with limited system efficiency degradation.

## 8. ACKNOWLEDGMENTS

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