Optimal Transmission Power Control in the Presence of a Smart Jammer

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Abstract—Jamming defense is an important yet challenging problem. In this paper, we study the jamming defense problem in the presence of a smart jammer, who can quickly learn the transmission power of the user and adaptively adjust its transmission power to maximize the damaging effect. By modeling the problem as a Stackelberg game, we compute the optimal transmission power for the user to maximize its utility, in spite of the existence of the smart jammer. We prove that the smart jammer is not more damaging than a jammer without the intelligence, provided that the user plays its strategy corresponding to a Stackelberg equilibrium. This nice property is due to the user’s ability to predict the jammer’s behavior.

I. INTRODUCTION

Jamming defense is an important yet challenging problem in wireless networks, since jamming attacks are easy to launch, as the jammer does not need any special hardware. Attacks of this kind usually aim at the physical layer and are realized by means of a high transmission power signal that corrupts a communication channel. We are interested in defending against smart jammers, who can quickly learn the transmission pattern of the users and adjust their jamming strategies so as to achieve maximum damage. As a first step along this line, we study the battle between a single user (a transmission-receiver pair) and a single smart jammer. Using a game theoretic approach, we derive optimal power control for the user in the presence of a smart jammer.

Game theory is a natural tool to solve this problem. Jamming defense can be considered as a game, where both the user and the jammer are players. A Nash Equilibrium (NE) is the status where no player has an incentive to change its strategy unilaterally so as to increase its own utility. Previous work [1–3] has been done on this topic by proving the existence of an NE and computing the NE. However, Nash Equilibrium is not the best solution to the problem studied in this paper, because the rationality of Nash Equilibrium is based on the assumption that all players take actions simultaneously. Whereas, in our model, the jammer is intelligent in the sense that it can quickly learn the user’s transmission power and adjust its transmission power accordingly.

To the best of our knowledge, this paper is the first to study the power control problem in the presence of a smart jammer. As an initial step, we consider a single user and a single jammer (the more challenging scenario with multi users/jammers is a subject of future research). The strategies of the user and the jammer are their transmission power levels. We consider the utilities of both the user and the jammer as functions of the SINR value. We model the power control problem with a smart jammer as a Stackelberg game [13], called Power Control with Smart Jammer (PCSJ) game. In this game, the user is the leader and the jammer is the follower. The user is aware of the jammer’s existence and has the knowledge of jammer’s intelligence, based on which the user chooses a certain strategy so as to maximize its own utility, while the jammer plays its optimal strategy given the user’s strategy. In this way, the user behaves like the so called ‘foresighted’ player, and the Stackelberg Equilibrium can be reached.

The rest of this paper is organized as follows: Section II briefly describes related work. The system model and the Stackelberg game formulation are introduced in Section III. In Section IV, we study the optimal strategies of the user and the jammer, leading to the optimal power control of the user. In Section V, we compare the Stackelberg Equilibrium with the Nash Equilibrium. In Section VI, we present numerical results. We conclude this paper in Section VII.
II. RELATED WORK

Due to the importance of jamming defense, wireless network jamming has been extensively studied in the past few years. A lot of jamming defense mechanisms on the physical layer have been proposed [7–9, 14, 15]. Mechanisms have been designed to detect jamming as well as avoid it. Frequency hopping and spread spectrum technology have been shown to be very effective to avoid jamming. With enough bandwidth or widely spread signals, it becomes harder to detect the start of a packet quickly enough in order to jam it. Some jamming-defense MAC protocols, proposed by Richa et al. ([10], [11]) also deal with the jamming problem: [10] proposed a protocol that works in multi-hop wireless networks modeled as a unit disk graph despite an adaptive adversary; [11] proposed a protocol that works in single-hop wireless networks despite a reactive adversary. Note that being reactive allows the adversary to make a jamming decision based on the actions of the nodes at the current step, which makes it more powerful.

Since jamming activities can be considered as a player (the jammer) playing against another player (the user), game theory is an appropriate method to deal with this kind of problem. Many previous works have studied jamming defense with game theory formulations [1, 3, 4, 6, 12]. In [6], Lai and El Gamal analyzed the resource allocation problem in fading multi-access channels using a Stackelberg game, where the base station is the leader and the users are followers. The base station first announces its strategy defined as the decoding order of the different users as a function of the channel state. Then the users compete based on this particular decoding strategy. In [1], Altman et al. studied the jamming game in wireless networks with transmission cost. In this game, both user and jammer take the power allocation on channels as their strategies. The utility function of the user is the weighted capacity minus transmission cost. The utility function of the jammer is the negative of the user’s weighted capacity minus transmission cost. They proved the existence and uniqueness of Nash Equilibrium. In addition, they provided analytical expressions for the equilibrium strategies. In [3], the same group of authors extended the jamming problem to the case with several jammers. The difference from [1] is that they did not consider transmission cost and they considered SINR and -SINR as the utility functions for user and jammer, respectively. They showed that the jammers equalize the quality of the best subcarriers for transmitter on as low level as their power constraint allows, mean while the user distributes his power among these jamming sub-carriers. In [12], Sagduyu et al. considered the power-controlled MAC game, which includes two types of players, selfish and malicious transmitters. Each type of user has different utility function depending throughput reward and energy cost. They also considered the case where the transmitters have incomplete information regarding other transmitter’s types, modeled as probabilistic beliefs. They derived the Bayesian Nash Equilibrium strategies for different degrees of uncertainty, and characterized the resulting equilibrium throughput of selfish nodes.

In all the previous works on jamming defense, the authors assume that the users and the jammers take actions simultaneously. In this paper, we study the power control problem in the presence of a smart jammer, which has more power compared to the jammer model studied before. To the best of our knowledge, we are the first to address this problem.

III. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we present the system model and formulate the problem to be studied.

A. System Model

Our system consists of a user (i.e., a transmitter-receiver pair), and a jammer (i.e., a transmitter), as illustrated in Figure 1. The user (jammer, respectively) has control over its own transmission power.

Let $P$ denote the transmission power of the user and $J$ denote the transmission power of the jammer. In addition, we assume that the user and the jammer transmit with cost $E$ and $C$ per unit power. As in [3, 12], we adopt SINR as the reward function of the user in our model. Hence, the utility function of the user is

$$u(P, J) = \frac{\alpha P}{N + \beta J} - EP, \quad (1)$$

and the utility function of the jammer is

$$v(P, J) = -\frac{\alpha P}{N + \beta J} - CJ, \quad (2)$$

where $N$ is the background noise level on the channel, and $\alpha > 0$ and $\beta > 0$ are fading channel gains of the user and the jammer, respectively.

In this paper, we deal with a smart jammer, which can quickly learn the user’s transmission power and adjust its transmission power accordingly to maximize (2). The user’s transmission power can be accurately estimated using physical carrier sensing and location knowledge. We are interested in determining the transmission power of the user such that the utility function (1) is maximized, in the presence of a smart jammer.

B. Stackelberg Game Formulation

We model the power control problem as a strategic game. In this game, both the user and the jammer are players. The strategy of each player is its transmission power. The objective of each player is to maximize its own utility function.
In our model, the jammer is smart. Based on this fact, we model the power control problem despite the smart jamming activity as a Stackelberg game, called Power Control with Smart Jammer (PCSJ) game. The Stackelberg game is an appropriate tool to model the scenario, where hierarchy of actions exists between players [5]. One of the players is the leader, and the others are followers. The leader chooses a strategy that can maximize its own utility, while each of the followers chooses a strategy that is its optimal strategy to the leader’s chosen strategy. Moreover, the leader can foresee the followers’ reaction, and hence adjust its strategy based on this knowledge. In our game, the user is the leader and the jammer is the follower. We assume that both of them have the knowledge of the channel information. In a Stackelberg game, the Stackelberg Equilibrium (SE) is the set of strategies of the players, such that the follower maximizes its utility given the leader’s strategy, and the leader maximizes its utility given that it knows the follower would do so.

We illustrate these concepts using a simple example given in Table I. Assume that Player A is the leader and Player B is the follower. If A plays strategy U, B would play strategy R, as it gives player B a utility of 5 (as opposed to a utility of 2 should B play strategy L). This leads to a utility of 6 for player A. If A plays strategy D, B would play strategy L, as it gives player B a utility of 3 (as opposed to a utility of 2 should B play strategy R). This leads to a utility of 4 for player A. Therefore A would play strategy U, leading to the Stackelberg equilibrium (U, R).

<table>
<thead>
<tr>
<th>Player B</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
<td>U</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>4.3</td>
</tr>
</tbody>
</table>

**TABLE I**
A simple game: The first number in each cell is the utility of Player A, while the second number is the utility of Player B.

IV. Optimal Transmission Power Control

In this section, we solve the power control problem by studying the PCSJ game. First, we compute the optimal strategy of the jammer, for a given strategy of the user. Then, we compute the optimal strategy of the user, based on the knowledge of the optimal strategy of the jammer.

A. Jammer’s Optimal Strategy

Assume that the user’s strategy $P$ is given. Then the jammer’s optimal strategy can be computed by solving the following optimization problem.

$$\max_{J \geq 0} v(P, J) = -\frac{\alpha P}{N + \beta J} - C J. \tag{3}$$

Thus we have the following lemma.

**Lemma 1.** Let $P$ be a given strategy of the user. Then the corresponding optimal strategy of the jammer is

$$J(P) = \begin{cases} 0, & P \leq \frac{CN^2}{\alpha \beta}, \\ \sqrt{\frac{\alpha CN^2}{\beta}} - N, & P > \frac{CN^2}{\alpha \beta}, \end{cases} \tag{4}$$

whereas the corresponding utility value of the jammer is

$$v(P, J(P)) = \begin{cases} -\frac{\alpha P}{N}, & P \leq \frac{CN^2}{\alpha \beta}, \\ \frac{CN - 2\sqrt{\alpha \beta CP}}{\beta}, & P > \frac{CN^2}{\alpha \beta}. \end{cases} \tag{5}$$

**Proof:** To find the maximum value of $v(P, J)$, we differentiate $v(P, J)$ with respect to $J$ and set the resulting derivative equal to 0,

$$0 = \frac{\partial v(P, J)}{\partial J} = \frac{\alpha \beta P}{(N + \beta J)^2} - C. \tag{6}$$

Considering the constraint $J \geq 0$, we have the optimal strategy of the jammer in (4). Plugging (4) into (2), we obtain the optimal value of jammer’s utility function in (5).

B. User’s Optimal Strategy

The user is aware of the existence of the jammer and knows that the jammer will play its optimal strategy to maximize its own utility. Therefore, the user can derive the jammer’s strategy based on Lemma 1. To compute the user’s optimal strategy, we solve the following optimization problem.

$$\max_{P \geq 0} u(P, J(P)) = \frac{\alpha P}{N + \beta J(P)} - EP, \tag{7}$$

where $J(P)$ is given in (4).

The optimal strategy and the optimal utility value are given in the following Lemma.

**Lemma 2.** The optimal strategy of the user is

$$P^{SE} = \begin{cases} \frac{\alpha C}{\sqrt{C N^2}}, & E \leq \frac{\alpha}{N}, \\ \frac{CN^2}{\alpha \beta}, & \frac{\alpha}{N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}, \end{cases} \tag{8}$$

and the optimal utility value of the user is

$$u(P^{SE}, J(P^{SE})) = \begin{cases} \frac{\alpha C}{\sqrt{C N^2}}, & E \leq \frac{\alpha}{N}, \\ \frac{\alpha E - N C N^2}{\alpha \beta}, & \frac{\alpha}{N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}. \end{cases} \tag{9}$$

**Proof:** Plugging (4) into the objective function (7), we have

$$u(P, J(P)) = \begin{cases} \left(\frac{\alpha}{N} - E\right) P, & P \leq \frac{CN^2}{\alpha \beta}, \\ \sqrt{\frac{\alpha CP}{\beta}} - EP, & P > \frac{CN^2}{\alpha \beta}. \end{cases} \tag{10}$$
Hence \( u(P, J(P)) \) is a linear function in \( P \) for \( 0 \leq P \leq \frac{CN^2}{\alpha \beta} \), and is a strictly concave function in \( P \) for \( P > \frac{CN^2}{\alpha \beta} \). Note that the derivative of \( u(P, J(P)) \) with respect to \( P \) in the range \( P > \frac{CN^2}{\alpha \beta} \) is given by

\[
\frac{\partial u(P, J(P))}{\partial P} = \frac{1}{2} \sqrt{\frac{\alpha C}{\beta P} - E}. \tag{11}
\]

Setting equation (11) to 0, we obtain \( P = \frac{\alpha C}{4 \beta E^2} \).

To compute the maximum value of (10), we consider three disjoint cases.

**Case-1**: \( E \leq \frac{\alpha}{2N} \). In this case, we can verify that \( \frac{CN^2}{\alpha \beta} \leq \frac{\alpha C}{4 \beta E^2} \). As illustrated in Figure 2(a), \( u(P, J(P)) \) achieves its maximum value of \( \frac{\alpha C}{4 \beta E^2} \) when \( P = \frac{\alpha C}{4 \beta E^2} \).

**Case-2**: \( \frac{\alpha}{2N} < E \leq \frac{\alpha}{N} \). In this case, we can verify that \( \frac{CN^2}{\alpha \beta} > \frac{\alpha C}{4 \beta E^2} \). As illustrated in Figure 2(b), \( u(P, J(P)) \) achieves its maximum value of \( \frac{(\alpha - EN)CN}{\alpha \beta} \) when \( P = \frac{CN^2}{\alpha \beta} \).

**Case-3**: \( E > \frac{\alpha}{N} \). In this case, we also have \( \frac{CN^2}{\alpha \beta} > \frac{\alpha C}{4 \beta E^2} \). As illustrated in Figure 2(c), \( u(P, J(P)) \) achieves its maximum value of 0 when \( P = 0 \).

This proves the lemma.

Lemmas 1 and 2 lead to the following theorem.

**Theorem 1.** The strategy pair \((P^{SE}, J^{SE})\) is the Stackelberg Equilibrium of the PCSJ game, where

\[
P^{SE} = \begin{cases} \frac{\alpha C}{4 \beta E^2}, & E \leq \frac{\alpha}{2N}, \\ \frac{CN^2}{\alpha \beta}, & \frac{\alpha}{2N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}, \end{cases}
\]

and

\[
J^{SE} = \begin{cases} \frac{\alpha - EN}{\alpha \beta}, & E \leq \frac{\alpha}{2N}, \\ \frac{\alpha}{2N}, & E > \frac{\alpha}{2N}. \end{cases}
\]

The utility of the user is

\[
u(P^{SE}, J^{SE}) = \begin{cases} \frac{\alpha C}{4 \beta E^2}, & E \leq \frac{\alpha}{N}, \\ \frac{(\alpha - EN)CN}{\alpha \beta}, & \frac{\alpha}{2N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}. \end{cases}
\]

and the utility of the jammer is

\[
u(P^{SE}, J^{SE}) = \begin{cases} \frac{\alpha}{\beta}, & E \leq \frac{\alpha}{2N}, \\ \frac{CN}{\beta}, & \frac{\alpha}{2N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}. \end{cases}
\]

**V. COMPARISON WITH NASH EQUILIBRIUM**

In this section, we study the impact of the jammer’s intelligence on the utilities of both players.

In the formulation of the PCSJ game, we have assumed that the jammer has the intelligence to quickly learn the transmission power of the user and adjust its own transmission power. If the jammer dose not have this intelligence, it would be natural to model the game in which both players play the game simultaneously in a non-cooperative manner. We refer the power control game, where the jammer is lack of the intelligence, as Power Control with Regular Jammer (PCRJ) game.

In the PCRJ game, a Nash Equilibrium (NE) strategy corresponds to a desirable strategy of the player, since by definition, it is a point where none of the players can improve its utility by unilaterally changing its strategy.

We show that the user’s utility at the SE of the PCSJ game is at least as high as the user’s utility at the NE of the PCRJ game, and the jammer’s utility at the SE of the PCSJ game is at least as high as the jammer’s utility at the NE of the PCRJ game.

Before getting to the proofs, we illustrate the concepts defined in this section using the example given in Table I. This time, we assume that players \( A \) and \( B \) play their strategies simultaneously. The strategy profile \((U, R)\) is not an NE, as player \( A \) can increase its utility from 6 to 8 by unilaterally changing its strategy from \( U \) to \( D \). \((D, L)\) is an NE for this example, because neither \( A \)
nor $B$ can increase its utility by unilaterally changing its strategy.

**Lemma 3.** There exists a unique NE $(P^{NE}, J^{NE})$ in the PCRJ game when the jammer does not have the intelligence to learn the user’s strategy. In addition,

$$(P^{NE}, J^{NE}) = \begin{cases} \left( \frac{\alpha C}{\beta E^2}, \frac{\alpha E - N}{\beta} \right), & E \leq \frac{N}{\alpha} \\ (0, 0), & E > \frac{N}{\alpha} \end{cases}$$

$$u(P^{NE}, J^{NE}) = 0,$$  

and

$$v(P^{NE}, J^{NE}) = \begin{cases} \frac{C}{\beta} \left( N - \frac{2\alpha}{E} \right), & E \leq \frac{N}{\alpha} \\ 0, & E > \frac{N}{\alpha} \end{cases}$$

**Proof:** We consider two disjoint cases:

**Case-1:** $E \leq \frac{N}{\alpha}$. If $J = \frac{\alpha E - N}{\beta}$, the value of (1) is 0, for any $P$. However, in order to have $J = \frac{\alpha E - N}{\beta}$, we must have $P = P^C$ according to (6). Thus the NE is $(P^{NE}, J^{NE}) = (P^C, \frac{\alpha E - N}{\beta})$.

We now prove the uniqueness of NE in this case. Assume to the contrary that there exists another NE $(P', J')$. We first use contradiction to prove that $J' = J^{NE}$. If $J' > J^{NE}$, $P' = 0$ according to (1). However, if $P' = 0$, we must have $J' = 0$ according to (2), contradicting the assumption that $J' > J^{NE} > 0$. If $J' < J^{NE}$, (1) becomes a strictly increasing function of $P$. Hence the user can increase its utility by unilaterally increase its transmission power, contradicting the NE assumption $(P', J')$. Thus we have proved that $J' = J^{NE}$. Since $J'$ is a function of $P'$ according to (6), we have $P' = P^{NE}$.

**Case-2:** $E > \frac{N}{\alpha}$. The derivative of (1) with respect to $P$ is

$$\frac{\alpha}{N + \beta J} - E < 0,$$

for any $J \geq 0$. Hence $P = 0$ is the unique optimal strategy for the user. Since $v(0, J) = -CJ$, $J = 0$ is the unique optimal strategy for the jammer. The NE is then $(P^{NE}, J^{NE}) = (0, 0)$.

Combining (12), (1) and (2), we get (13) and (14).

Lemma 3 leads to the following important theorem, as stated in the 3rd paragraph of this section.

**Theorem 2.** Let $(P^{SE}, J^{SE})$ be the SE of the PCSJ game, and $(P^{NE}, J^{NE})$ be the NE of the PCRJ game. Then we have

$$u(P^{SE}, J^{SE}) \geq u(P^{NE}, J^{NE}),$$

$$v(P^{SE}, J^{SE}) \geq v(P^{NE}, J^{NE}).$$

**Proof:** It is clear that $u(P^{SE}, J^{SE}) \geq u(P^{NE}, J^{NE})$. For the jammer, it suffices to prove that $v(P^{SE}, J^{SE}) \geq v(P^{NE}, J^{NE})$ when $\frac{N}{2\alpha} < E \leq \frac{N}{\alpha}$.

According to (14), we have

$$v(P^{NE}, J^{NE}) = \frac{C}{\beta} \left( N - \frac{2\alpha}{E} \right) \leq \frac{C}{\beta} (N - 2N) = -\frac{CN}{\beta} = v(P^{SE}, J^{SE})$$

The theorem is proved.

**VI. SIMULATIONS**

In order to validate our theoretical insights, we ran simulations and computed the SE of the PCSJ game, as well as utilities of the user and the jammer at the SE. For comparison, we also computed the NE of the PCRJ game, as well as the utilities at the NE. The simulation results show that SE leads to higher utilities for both players than NE.

Five variables determine the players’ strategies and their utilities, which are $N$, $\alpha$, $\beta$, $C$, and $E$. Among these five variables, only $\alpha$ and $\beta$, i.e., fading channel gains of the user and the jammer, may vary significantly due to the change of players’ physical locations. Hence, we explore the relations of user and jammer’s utilities with respect to different values of $\alpha$ and $\beta$. We set $\alpha$ and $\beta$ to be in the range of $[0.1, 0.9]$. Moreover, let $C = E = 0.1$ (as in [1]), and $N = 2$.

Figures 3(a) and 3(b) show the impact of $\alpha$ on the players’ utilities, with $\beta = 0.5$. We observe that SE leads to higher utilities for both players than NE does. Recall that $\alpha$ is the fading channel gain of the user. Therefore the larger $\alpha$ is, the closer the transmitter is from the receiver. Hence, as $\alpha$ increases, user’s SE utility increases while jammer’s SE utility decreases.

Figures 4(a) and 4(b) show the impact of $\beta$ on the players’ utilities, with $\alpha = 0.5$. Again, SE leads to higher
utilities for both players than NE does. Note that the jammer’s SE utility increases while the user’s SE utility decreases due to the fact that the jammer’s influence on the receiver gets stronger as $\beta$ increases.

![User's Utility](image1)

![Jammer's Utility](image2)

Fig. 4. Impact of $\beta$ on players’ utilities

We can make many other observations. For example, we observe from Figure 3(a) and Figure 4(a) that the utility of the user is 0 in the NE. This validates our claim in Lemma 3. We can also observe from Figure 3(a) that the user’s SE utility is 0 for $\alpha \leq 0.2$, and positive for $\alpha > 0.2$. This confirms our analysis in Theorem 1 and Lemma 2, which says the user’s SE utility is 0 when $\alpha < EN$, and $(\alpha - EN)CN$ when $EN \leq \alpha < 2EN$. Note that we have $EN = 0.1 \times 2 = 0.2$ for Figure 3(a).

VII. CONCLUSIONS

In this paper, we have studied the problem of optimal power control in the presence of a smart jammer, who can quickly learn the transmission power of the user and adjust its transmission power to maximize the damaging effect. By modeling the problem as a Stackelberg game, we have derived the optimal power control strategy for the user to maximize its utility, despite the existence of the smart jammer. We have proved that, due to the user’s ability to predict the jammer’s behavior, the smart jammer is not more damaging than a jammer without the intelligence.

This paper studied the single-user single-jammer single-channel scenario. As future research directions, it would be interesting to consider the scenarios, where there are multiple users or multiple channels.

REFERENCES


