Two-Tiered Constrained Relay Node Placement in Wireless Sensor Networks: Computational Complexity and Efficient Approximations

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Abstract—In wireless sensor networks, relay node placement has been proposed to improve energy efficiency. In this paper, we study two-tiered *constrained* relay node placement problems, where the relay nodes can be placed only at some pre-specified candidate locations. To meet the connectivity requirement, we study the *connected single-cover* problem where each sensor node is *covered* by a base station or a relay node (to which the sensor node can transmit data), and the relay nodes form a connected network with the base stations. To meet the survivability requirement, we study the 2-*connected double-cover* problem where each sensor node is *covered* by two base stations or relay nodes, and the relay nodes form a 2-connected network with the base stations. We study these problems under the assumption that $R \geq 2r > 0$, where R and r are the communication ranges of the relay nodes and the sensor nodes, respectively. We investigate the corresponding computational complexities, and propose novel polynomial time approximation algorithms for these problems. Specifically, for the connected single-cover problem, our algorithms have $\mathcal{O}(1)$-approximation ratios. For the 2-connected double-cover problem, our algorithms have $\mathcal{O}(1)$-approximation ratios for practical settings and $\mathcal{O}(\ln n)$-approximation ratios for arbitrary settings. Experimental results show that the number of relay nodes used by our algorithms is no more than twice of that used in an optimal solution.

Index Terms—Relay node placement, wireless sensor networks, connectivity and survivability.

1 INTRODUCTION

In wireless sensor networks (WSNs), many low-cost and low-power *sensor nodes* (SNs)[1] sense the environment and transmit sensed information over possibly multiple hops to the base stations (BSs). Since energy conservation is a primary concern in WSNs, there has been extensive research on energy aware routing [5, 14, 20, 35]. To prolong network lifetime while meeting certain network specifications, researchers have proposed to deploy a small number of *relay nodes* (RNs) in a WSN to communicate with the SNs, BSs, and other RNs [2, 3, 6, 9, 13, 16, 18, 22, 23, 34]. This is studied under the theme of *relay node placement*.

In general, the relay node placement problem has been studied from two perspectives, namely the routing structure and the connectivity requirements. The study based on the routing structure may be further classified into *single-tiered* and *two-tiered* [9, 13, 23, 27]. In single-tiered relay node placement, the SNs may also forward packets. In two-tiered relay node placement, the SNs transmit their sensed data to an RN or a BS, but do not forward packets for other nodes. The two-tiered network is essentially a cluster-based network, where each RN acts as the cluster head in the corresponding cluster. Extensive research efforts have been done on cluster-based networks, for example, energy-efficient cluster-based protocol designed by Heinzelman *et al.* [14], topology control presented by Pan *et al.* [27], and clustering algorithms proposed by Gupta and Younis [11], and Younis and Fahmy [35]. The study based on connectivity requirements can be classified into *connected* and *survivable* [2, 3, 13, 18, 36]. For connected relay node placement, the placement of RNs ensures the connectivity between the SNs and the BSs. For survivable relay node placement, the placement of RNs ensures the biconnectivity between the SNs and the BSs.

Most of the previous works [2, 3, 6, 9, 12, 13, 18, 22, 23, 32, 36] study *unconstrained* relay node placement, in the sense that the RNs can be placed *anywhere*. In practice, however, there may be some physical constraints on the placement of the RNs: e.g., there may be a lower bound on the distance between two nodes to reduce interference; or there may be some forbidden regions where relay nodes cannot be placed. For instance, in a WSN application

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for monitoring an active volcano that is ready to erupt, the inside of the crater (where the temperature could reach many thousand degrees) is a forbidden region for the placement of relay nodes (any nodes). In a nutshell, there are many such practical applications where the placement in general is constrained.

The relay node placement problem subject to forbidden regions and the lower bound on internode distances, is intrinsically more challenging, compared to its unconstrained counterpart. Taking an initial step towards solving this challenging problem, we study constrained relay node placement problems where the RNs can only be placed at a set of candidate locations. Our formulation can be viewed as an approximation to the aforementioned relay node placement problem, subject to the constraints of forbidden regions and the lower bound on internode distance in the following sense. Instead of allowing the relay nodes to be placed anywhere outside of the forbidden regions and satisfying the internode distance bound, we further restrict the placement of the relay nodes to certain candidate locations that are outside of the forbidden regions. The use of candidate locations simultaneously approximates the constraints enforced by the forbidden regions and the internode distance bound.

1.1 Main Contributions

In this paper, we study the two-tiered constrained relay node placement problem, under both connectivity and survivability requirements. As discussed earlier, the two-tiered problem strongly resembles the real world deployment of WSNs. Our main contributions are summarized as follows:

- To the best of our knowledge, this is the first work to study constrained relay node placement in the two-tiered model. The model under consideration assumes the usage of base stations, but it is straightforward to extend our algorithms to the cases without base stations. Furthermore, it is assumed that the communication range of the relay nodes is at least twice of that of the sensor nodes, without which the approximations of our algorithms may not hold, but this assumption has been shown to be valid for most scenarios of HWSNs [13, 32] and adopted in [25, 36] as well.
- We study the relay node placement problem with connectivity requirement, named the *connected single-cover problem* (1CSCP). In this problem, our objective is to place a minimum number of RNs such that (1) each SN is covered by at least one BS or RN and (2) the RNs form a connected network with the BSs. We formulate the problem, prove its NP-hardness, and present a polynomial time $\mathcal{O}(1)$-approximation algorithm.
- We study the relay node placement problem with survivability requirement, named the 2-*connected double-cover problem* (2CDCP). In this problem, we intend to place a minimum number of RNs such that (1) each SN is covered by at least two BSs or RNs and (2) the RNs form a 2-connected network with the BSs. We formulate the problem, study its computational complexity, and present polynomial time approximation algorithms. Our algorithms have $\mathcal{O}(1)$-approximation ratios for practical settings, where there is a constant bound on the number of candidate RN locations each SN can be connected to, and $\mathcal{O}(\ln n)$-approximation ratios for arbitrary settings.

1.2 Paper Organization

The rest of the paper is organized as follows. In Section 2, we provide a brief literature survey. In Section 3, we present the preliminary definitions and lemmas. In Section 4, we study 1CSCP. In Section 5, we study 2CDCP. Section 6 presents linear programming formulations for efficiently computing lower bounds on the optimal solutions of the relay node placement problems. We present experimental results in Section 7 and conclude the paper in Section 8.

2 Related Work

In this section, we briefly review the related work. Throughout, we will use r and R to denote the communication ranges of SNs and RNs, respectively. We will use $k = 1$ to denote connectivity requirement and $k \geq 2$ to denote survivability requirement. The problems studied in this paper fall under the two-tiered classification. For the single-tiered classification, we refer interested readers to [2, 3, 6, 9, 12, 13, 18, 21–23, 25, 32, 36].

The two-tiered relay node placement problem has been studied by [9, 12, 13, 22, 23, 32, 36]. The two-tiered model applies well to the highly scalable clustered wireless sensor networks [11], wherein the RNs act as the cluster heads [11, 27, 34]. In the two-tiered problems formulated by Hao *et al.* [13], an SN has to connect to at least 2 RNs and the RNs need to form a 2-connected network, under the assumption that $R \geq r$. Sometimes, the SNs may be assigned different power levels to minimize the total power consumption [4]. In [12], Han *et al.* studied relay node placement in a heterogeneous WSN, where the SNs have different transmission radii. They presented $\mathcal{O}(1)$-approximation algorithms for given $k \geq 2$. Liu *et al.* [22] proposed two $\mathcal{O}(1)$-approximation algorithms for the problem with $R = r$ and $k = 2$. Lloyd and Xue [23] studied the problem for $R \geq r$ and $k = 1$, and proposed a $(5 + \epsilon)$ algorithm, where ϵ is any positive constant. Recently, Efrat *et al.* [9] improved it to PTAS [8]. Srinivas *et al.* [32] studied the problem of the maintenance of a mobile backbone using the minimum number of backbone nodes given that $R \geq 2r$ and $k = 1$. Zhang *et al.* [36] studied the problem with $R \geq r$ and $k = 2$ and presented a $(20 + \epsilon)$-approximation algorithm when the BSs are considered. To the best of our knowledge, our paper is the first work on two-tiered constrained relay node placement.

3 Definitions and Preliminaries

In this section, we formally define the problems and present some basic lemmas to be used in later sections. We consider a heterogeneous wireless sensor network (HWSN) consisting of three kinds of nodes: *base stations* (BSs), *sensor nodes* (SNs), and *relay nodes* (RNs). We are interested in the scenario where RNs can be placed only at certain candidate locations. Throughout this paper, we use $\mathcal{B}$ to denote the set of base stations, $\mathcal{S}$ to denote the set of sensor nodes, and $\mathcal{X}$ to denote the set of candidate locations where relay nodes can be placed. We make the following assumptions as in [13, 25, 32]. For given constants $R \geq 2r > 0$, the communication range of the SNs is r, that of the RNs is R, and that of the BSs is much greater than R such that any two BSs can communicate directly with each other (as they are connected, e.g., via either the wired networks or the satellites). Note that the assumption $R \geq 2r$ is valid for most scenarios of HWSNs [13, 32]. For example, TelosB motes (possible sensor nodes) have an outdoor transmission range between 75m and 100m; IRIS motes (possible relay nodes) have an outdoor transmission range about 300m; IEEE 802.11 routers (possible relay nodes) have an outdoor transmission range about 800m. We use $d(x, y)$ to denote the Euclidean distance between two points x and y in the plane. We also use u to denote the location of a node u, where it is unambiguous.

We are interested in a two-tiered set-up, where the SNs send the sensed data to an RN or a BS within distance r, while the RNs can forward received packets to an RN or a BS within distance R. We define the *hybrid communication graph* (HCG) and the *relay communication graph* (RCG) in the following.

Definition 3.1: [Hybrid Communication Graph] Let r, R, $\mathcal{B}$, $\mathcal{S}$, and $\mathcal{X}$ be given. Let $\mathcal{R}$ be a subset of $\mathcal{X}$. The *hybrid communication graph* $HCG(r, R, \mathcal{B}, \mathcal{S}, \mathcal{R})$ induced by the 5-tuple $(r, R, \mathcal{B}, \mathcal{S}, \mathcal{R})$ is a *directed graph* with vertex set $V = \mathcal{B} \cup \mathcal{S} \cup \mathcal{R}$ and edge set E defined as follows. For any two BSs $b_i, b_j \in \mathcal{B}$, E contains the *bi-directed edge* (b_i, b_j). For an RN $y \in \mathcal{R}$ and a node $x \in \mathcal{B} \cup \mathcal{R}$, E contains the *bi-directed edge* (y, x) if and only if $d(y, x) \leq R$. For an SN $s \in \mathcal{S}$ and a node $x \in \mathcal{R} \cup \mathcal{B}$, E contains the *directed edge* (s, x) if and only if $d(s, x) \leq r$. □

Definition 3.2: [Relay Communication Graph] Let R, $\mathcal{B}$, and $\mathcal{R}$ be given as in Definition 3.1. The *relay communication graph* $RCG(R, \mathcal{B}, \mathcal{R})$ induced by the 3-tuple $(R, \mathcal{B}, \mathcal{R})$ is an *undirected graph* with vertex set $V = \mathcal{B} \cup \mathcal{R}$ and edge set E defined as follows. For any two BSs $b_i, b_j \in \mathcal{B}$, E contains the *undirected edge* (b_i, b_j). For an RN $y \in \mathcal{R}$ and a node $x \in \mathcal{B} \cup \mathcal{R}$, E contains the undirected edge $(y, x) = (x, y)$ if and only if $d(y, x) \leq R$. □

The HCG characterizes all possible pairwise communications between pairs of nodes. The RCG characterizes all possible communications between the RNs and/or BSs. We also define in the following the concepts related to the *weight* and the *relay size* of an RCG. These concepts will be used later to establish relationships between the weight of a subgraph and the number of corresponding RNs. We use the following standard graph-theoretic notations: for a graph G, $V(G)$ denotes the vertex set of G and $E(G)$ denotes the edge set of G.

Definition 3.3: Let $G = RCG(R, \mathcal{B}, \mathcal{X})$ be a relay communication graph. Let $\mathcal{R}$ be a given subset of $\mathcal{X}$. For each edge $e = (u, v)$ in the RCG, we define its *weight* induced by $\mathcal{R}$ (denoted by $w^{\mathcal{R}}(e)$) as

$$w^{\mathcal{R}}(e) = |\{u, v\} \cap (\mathcal{X} \setminus \mathcal{R})|. \tag{1}$$

Let $\mathcal{H}$ be a subgraph of G. The weight of $\mathcal{H}$ (denoted by $w^{\mathcal{R}}(\mathcal{H})$) is defined as

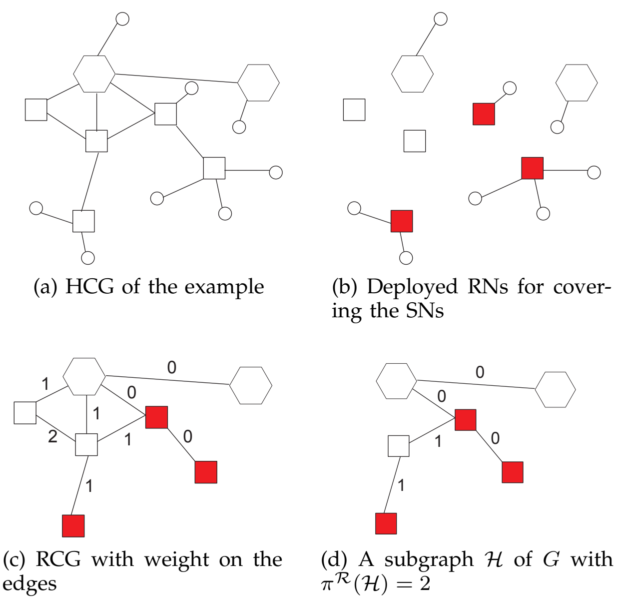
$$w^{\mathcal{R}}(\mathcal{H}) = \sum_{e \in E(\mathcal{H})} w^{\mathcal{R}}(e). \tag{2}$$

The relay-size of $\mathcal{H}$ (denoted by $\pi^{\mathcal{R}}(\mathcal{H})$) is defined as

$$\pi^{\mathcal{R}}(\mathcal{H}) = |V(\mathcal{H}) \cap (\mathcal{X} \setminus \mathcal{R})|. \tag{3}$$

We call e a *weight-0* edge if $w^{\mathcal{R}}(e) = 0$, a *weight-1* edge if $w^{\mathcal{R}}(e) = 1$, and a *weight-2* edge if $w^{\mathcal{R}}(e) = 2$. □

The example in Fig. 1 illustrates Definition 3.3.



(a) HCG of the example

(b) Deployed RNs for covering the SNs

(c) RCG with weight on the edges

(d) A subgraph $\mathcal{H}$ of G with $\pi^{\mathcal{R}}(\mathcal{H}) = 2$

Fig. 1: An example illustrating the concepts in Definition 3.3. The hexagons represent the BSs, the circles represent the SNs, and the squares represent the candidate RN locations, of which the solid ones represent the deployed RNs for covering the SNs.

Lemma 3.1: Let $\mathcal{H}$ be a subgraph of $RCG(R, \mathcal{B}, \mathcal{X})$ and $\mathcal{R} \subseteq \mathcal{X}$ be a selected set of RNs. Assume that each $RN \in \mathcal{X} \setminus \mathcal{R}$ in $\mathcal{H}$ has degree at least 2 (in $\mathcal{H}$), then $w^{\mathcal{R}}(\mathcal{H}) \geq 2 \cdot \pi^{\mathcal{R}}(\mathcal{H})$. □

PROOF. This lemma can be proved by *shifting the weight of an edge to its end nodes*. We refer the reader to Lemma 2.1 in [25], while noting the following differences. Instead of the HCG we consider an RCG here, and instead of all RNs we consider only those belonging to $\mathcal{X} \setminus \mathcal{R}$. ■

We will also need the results stated in Lemma 3.2 and Lemma 3.3. These lemmas have been proved by Misra *et al.* in [25, Lemma 2.2 and Lemma 2.3], we state them here without the proofs.

Lemma 3.2: Let $G(V, E)$ be an undirected 2-connected graph where $|V| \geq 3$ and each edge $e \in E$ has a unit length $l(e) = 1$. Let $\mathcal{H}(V, E')$ be a minimum length 2-connected subgraph of G. Then $|E'| \leq 2|V| - 3$. □

Lemma 3.3: Let $G(V, E)$ be an undirected connected graph where $|V| \geq 3$ and each edge $e \in E$ has a unit length $l(e) = 1$. Let $\mathcal{H}(V, E')$ be a minimum length connected subgraph of G such that two vertices u and v are in the same 2-connected component of $\mathcal{H}$ if and only if they are in the same 2-connected component of G. Then $|E'| \leq 2|V| - 1$. □

As is standard, *a polynomial time α-approximation algorithm* for a minimization problem is an algorithm $\mathbb{A}$ that, for any instance of the problem, computes a solution that is at most α times the optimal solution to the instance, in time bounded by a polynomial in the input size of the instance [8]. Approximation algorithm $\mathbb{A}$ is also said to have an *approximation ratio of* α. For graph-theoretic terms not defined in this paper, we refer readers to the standard textbook [33]. The terms *nodes* and *vertices* are used interchangeably, as well as terms *links* and *edges*. For concepts in algorithms and the theory of computation, we refer readers to the standard textbooks [8, 10].

4 CONNECTED SINGLE-COVER

In this section, we study the connected single-cover problem. Refer to the first paragraph in Section 3 for the notations r, R, $\mathcal{B}$, $\mathcal{S}$, and $\mathcal{X}$.

4.1 Problem Definitions and Notation

Definition 4.1: [SCP] A subset $\mathcal{R} \subseteq \mathcal{X}$ is called a *single-cover* (denoted by SC) of $\mathcal{S}$ if every SN in $\mathcal{S}$ is within distance r of $\mathcal{R} \cup \mathcal{B}$. The *size* of the corresponding SC is $|\mathcal{R}|$. An SC of $\mathcal{S}$ is called a *minimum single-cover* (denoted by MSC) of $\mathcal{S}$ if it has the minimum size among all SCs of $\mathcal{S}$. The *single-cover relay node placement problem* (denoted by SCP$(r, \mathcal{B}, \mathcal{S}, \mathcal{X})$) for $(r, \mathcal{B}, \mathcal{S}, \mathcal{X})$ seeks an MSC of $\mathcal{S}$. □

The SCP is closely related to the *geometric disk hitting problem* (GDHP), defined formally in the following.

Definition 4.2: [GDHP] Given a set $\mathcal{P}$ of points and a set $\mathcal{D}$ of disks with radius r. A subset $\mathcal{P}' \subseteq \mathcal{P}$ is called a *hitting set* if any disk in $\mathcal{D}$ is *hit* by at least one point in $\mathcal{P}'$, that is, the center of any disk is within distance r of at least one point in $\mathcal{P}'$. The *geometric disk hitting problem* (denoted by GDHP$(r, \mathcal{D}, \mathcal{P})$) seeks a hitting set of minimum size. □

Definition 4.3: [1CSCP] A subset $\mathcal{R} \subseteq \mathcal{X}$ is called a *connected single-cover* (denoted by 1CSC) of $\mathcal{S}$ if (a) $\mathcal{R}$ is a single-cover of $\mathcal{S}$ and (b) the relay communication graph $RCG(R, \mathcal{B}, \mathcal{R})$ is connected. The *size* of the corresponding 1CSC is $|\mathcal{R}|$. A 1CSC is called a *minimum connected single-cover* (denoted by M1CSC) of $\mathcal{S}$ if it has the minimum size among all 1CSCs of $\mathcal{S}$. The *connected single-cover relay node placement problem* (denoted by 1CSCP) for $(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ seeks an M1CSC of $\mathcal{S}$. □

Observe that existing studies [2, 6, 16, 18, 23, 32, 36] considered unconstrained relay node placement. In contrast, our study here focuses on the connected single cover problem subject to constraints. The solutions to some of the unconstrained problems may deploy an RN on top of an SN or another RN. In practice, there are some physical constraints on the RN deployment. For example, there should be a minimum distance between two RNs to reduce interference. There may also be some forbidden regions where RNs cannot be deployed. It is clear that our model is more practical than previous models. This work also differs from the work in [25], because we are studying a two-tiered network while [25] studied the single-tiered problem.

4.2 Computational Complexity

Before designing algorithms for 1CSCP, we prove that 1CSCP is NP-hard. We first prove that SCP is NP-hard by a reduction from GDHP, which is known to be NP-hard [24]. We then prove that 1CSCP is NP-hard by a reduction from SCP.

Lemma 4.1: SCP is NP-hard. □

PROOF. We prove this by reduction from GDHP. Let an instance $\mathcal{I}_1$ of GDHP be given by $(r, \mathcal{D}, \mathcal{P})$. An instance $\mathcal{I}_2$ of SCP is given by $(r, \mathcal{B}, \mathcal{S}, \mathcal{X})$ where the sensor transmission range r is the same as in $\mathcal{I}_1$; the set of sensor nodes $\mathcal{S}$ is set to the centers of the disks in $\mathcal{D}$; the set of relay candidate locations $\mathcal{X}$ is set to $\mathcal{P}$; and the set of base stations $\mathcal{B}$ consists of a single point b that is not in any of the disks in $\mathcal{D}$. This construction takes polynomial time. Moreover, a subset of $\mathcal{P}$ is an optimal solution to $\mathcal{I}_1$ if and only it is an optimal solution to $\mathcal{I}_2$. This completes the proof. ■

Using the techniques in the proof of Lemma 4.1, we can also construction a reduction from SCP to GDHP. More importantly, an α-approximation algorithm for GDHP can be used as an α-approximation algorithm for SCP and vice versa.

Theorem 4.1: 1CSCP is NP-hard. □

PROOF. Let an instance $\mathcal{I}_1$ of SCP be given by $(r, \mathcal{B}, \mathcal{S}, \mathcal{X})$, where $\mathcal{B} = \{b_1, \ldots, b_B\}$, $\mathcal{S} = \{s_1, \ldots, s_n\}$, $\mathcal{X} = \{x_1, \ldots, x_t\}$, and $r > 0$. We construct an instance $\mathcal{I}_2$ of 1CSCP by $(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$, where $R = \max_{1 \leq i < j \leq t} ||x_i - x_j||$. This results in the subgraph $G(R, \mathcal{X})$ of the HCG to be a complete graph.

It is easy to see that a subset $\mathcal{X}' \subseteq \mathcal{X}$ is an optimal solution to $\mathcal{I}_1$ if and only if it is an optimal solution to $\mathcal{I}_2$. Therefore, 1CSCP is NP-hard. ■

4.3 A Framework of Efficient Approximation Algorithms for 1CSCP

In this subsection, we present a framework of polynomial time approximation algorithms for 1CSCP. The framework decomposes 1CSCP into two subproblems, outlined as follows. It first applies an algorithm $\mathbb{A}$ for SCP to compute a single-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$ with a small cardinality. It then applies another algorithm $\mathbb{B}$ for the Steiner tree problem [17] to augment $\mathcal{R}_{\mathbb{A}}$ by some other RNs $\mathcal{R}_{\mathbb{B}}$ so that the relay communication graph $RCG(R, \mathcal{B}, \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{\mathbb{B}})$ is connected. The union of $\mathcal{R}_{\mathbb{A}}$ and $\mathcal{R}_{\mathbb{B}}$ is output as a connected single-cover of $\mathcal{S}$.

Algorithm 1 Approximation for 1CSCP$(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$

Input: Set $\mathcal{B}$ of BSs, set $\mathcal{S}$ of SNs, set $\mathcal{X}$ of candidate RN locations, sensor node communication range $r > 0$, and relay node communication range $R \geq 2r > 0$. An approximation algorithm $\mathbb{A}$ for SCP, and an approximation algorithm $\mathbb{B}$ for the Steiner tree problem.

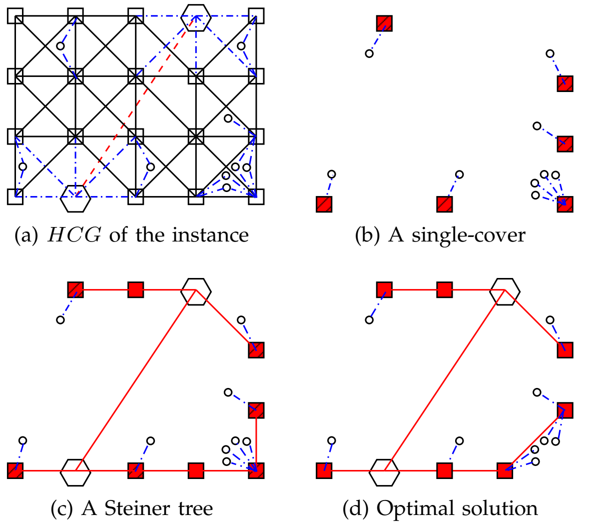
Output: A connected single-cover $\mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{\mathbb{B}} \subseteq \mathcal{X}$.

1: Remove from $\mathcal{S}$ all SNs within distance r from $\mathcal{B}$. Apply algorithm $\mathbb{A}$ to obtain a single-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$. Without loss of generality, we assume that $\mathcal{R}_{\mathbb{A}}$ is *minimal*, meaning that none of its proper subsets is a single-cover of $\mathcal{S}$.
2: Construct the relay communication graph $G = RCG(R, \mathcal{B}, \mathcal{X})$.
3: **if** the nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$ are not in the same connected component of $RCG(R, \mathcal{B}, \mathcal{X})$ **then**
4: Stop. The given instance of 1CSCP does not have a feasible solution.
5: **end if**
6: Assign *edge weight* in G as (1), with $\mathcal{R}$ substituted by $\mathcal{R}_{\mathbb{A}}$, i.e., for an edge (x, y) in G, we have

$$w^{\mathcal{R}_{\mathbb{A}}}(x, y) = |(\mathcal{X} \setminus \mathcal{R}_{\mathbb{A}}) \cap \{x, y\}|.$$

7: Apply algorithm $\mathbb{B}$ to compute a low weight Steiner tree [17] $\mathcal{T}_{\mathbb{B}}$ of $RCG(R, \mathcal{B}, \mathcal{X})$, spanning the node set $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$. Let the set of Steiner points in the tree $\mathcal{T}_{\mathbb{B}}$ be $\mathcal{R}_{\mathbb{B}}$.
8: Output $\mathcal{R}_{\mathbb{A},\mathbb{B}} = \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{\mathbb{B}}$.

We illustrate the major steps of Algorithm 1 via the example shown in Fig. 2. The instance has 2 base stations (hexagons), 9 sensor nodes (circles), and 18 candidate RN locations (squares). We also have $R = 2r$. The nodes, as well as the hybrid communication graph $HCG(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ are shown in Fig. 2(a), where we have omitted the directions of the SN-RN edges for clarity. Algorithm 1 performs the following



(a) *HCG* of the instance (b) A single-cover (c) A Steiner tree (d) Optimal solution

Fig. 2: (a) The *HCG* for 2 BSs (hexagons), 9 SNs (circles), and 18 candidate RN locations (squares). (b) A feasible solution to SCP. (c) The resulting 1CSC, which uses 8 RNs. (d) The optimal solution, which uses 7 RNs.

steps. In Line 1, we obtain a single-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$, which contains 6 relay nodes, shown in Fig. 2(b). In Line 2, we construct the RCG, which is obtained by deleting from the HCG in Fig. 2(a) all edges incident from a sensor node. In Lines 3-5, we find that the nodes in $\mathcal{R}_{\mathbb{A}} \cup \mathcal{B}$ are in the same connected component. In Line 6, we assign each edge a weight following (1). In Line 7, we obtain a Steiner tree spanning the node set $\mathcal{R}_{\mathbb{A}} \cup \mathcal{B}$, shown in Fig. 2(c). Here we used the MST-based approximation algorithm [19] to obtain the Steiner tree. The Steiner tree has two Steiner nodes, leading to the use of two more relay nodes: $|\mathcal{R}_{\mathbb{B}}| = 2$. Fig. 2(d) shows an M1CSC, which uses 7 relay nodes. The solution produced by Algorithm 1 uses 8 relay nodes, which is not optimal, but close to optimal.

Theorem 4.2: The worst case running time of Algorithm 1 is $\mathcal{O}(T_{\mathbb{A}} + T_{\mathbb{B}} + |\mathcal{B} \cup \mathcal{X}|^2)$, where $T_{\mathbb{A}}$ and $T_{\mathbb{B}}$ are the time complexities of the approximation algorithms $\mathbb{A}$ and $\mathbb{B}$, respectively. Furthermore, we have:

(a) $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ has a feasible solution if and only if $\mathcal{S}$ has a single-cover $\mathcal{R} \subseteq \mathcal{X}$ such that all nodes in $\mathcal{B} \cup \mathcal{R}$ are in the same connected component of $RCG(R, \mathcal{B}, \mathcal{X})$.

(b) When $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ has a feasible solution, Algorithm 1 guarantees computing a connected single-cover $\mathcal{R}_{\mathbb{A},\mathbb{B}}$ of $\mathcal{S}$, which uses no more than $(\alpha + 3.5\beta)$ times the number of RNs required in an optimal solution $\mathcal{R}_{opt}$ for $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$, where α and β are the approximation ratios of $\mathbb{A}$ and $\mathbb{B}$, respectively. □

PROOF. Line 1 of Algorithm 1 computes a single-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$ in $T_{\mathbb{A}}$ time. Line 2 constructs the RCG in $\mathcal{O}(|\mathcal{B} \cup \mathcal{X}|^2)$ time. Using depth first search, Lines

3-5 can be accomplished in $\mathcal{O}(|\mathcal{B} \cup \mathcal{X}|^2)$ time. Line 6 assigns weights to the edges in the RCG, which requires $\mathcal{O}(|\mathcal{B} \cup \mathcal{X}|^2)$ time. Line 7 requires $T_{\mathbb{B}}$ time. This proves the time complexity of the algorithm.

The correctness of (a) follows from the definition of 1CSCP. Therefore we only need to prove the correctness of (b).

Roadmap. We first show that Algorithm 1 is guaranteed to find a connected single-cover when the instance is feasible. We then prove the approximation ratio of Algorithm 1, i.e., $|\mathcal{R}_{\mathbb{A},\mathbb{B}}| \leq (\alpha + 3.5\beta)|\mathcal{R}_{opt}|$. To prove the ratio, we prove $|\mathcal{R}_{\mathbb{A}}| \leq \alpha|\mathcal{R}_{opt}|$ and $|\mathcal{R}_{\mathbb{B}}| \leq 3.5\beta|\mathcal{R}_{opt}|$, respectively. The ratio then follows from the fact that $|\mathcal{R}_{\mathbb{A},\mathbb{B}}| = |\mathcal{R}_{\mathbb{A}}| + |\mathcal{R}_{\mathbb{B}}|$.

The feasibility of the instance ensures that Line 1 of Algorithm 1 can find a single-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$. Let $\mathcal{R}_{opt}$ be an optimal solution to $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$. Then $\mathcal{R}_{opt}$ is a single-cover of $\mathcal{S}$, and $RCG(R, \mathcal{B}, \mathcal{R}_{opt})$ is connected. Since $\mathcal{R}_{\mathbb{A}}$ is a minimal single-cover of $\mathcal{S}$, each node $y \in \mathcal{R}_{\mathbb{A}}$ is within distance r of an SN $s \in \mathcal{S}$ and s is not within distance r of $\mathcal{B}$. Note that s must be within distance r of a node $y' \in \mathcal{R}_{opt}$, as $\mathcal{R}_{opt}$ is a single-cover of $\mathcal{S}$. Therefore y is either in $\mathcal{R}_{opt}$ or within distance $r + r \leq R$ of a node (y' for example) in $\mathcal{R}_{opt}$. Therefore $RCG(R, \mathcal{B}, \mathcal{R}_{opt} \cup \mathcal{R}_{\mathbb{A}})$ is also connected. Hence Line 7 of Algorithm 1 is guaranteed to find a Steiner tree, thereby producing a connected single-cover $\mathcal{R}_{\mathbb{A},\mathbb{B}}$ for the instance of $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$.

In the following, we will prove that $|\mathcal{R}_{\mathbb{A},\mathbb{B}}|$ is no more than $(\alpha + 3.5\beta)|\mathcal{R}_{opt}|$. Let $\mathcal{R}_{msc}$ be a minimum single-cover for the given instance. Then we must have $|\mathcal{R}_{msc}| \leq |\mathcal{R}_{opt}|$, as every connected single-cover is a single-cover. Since $\mathbb{A}$ has approximation ratio α, we have

$$|\mathcal{R}_{\mathbb{A}}| \leq \alpha|\mathcal{R}_{msc}| \leq \alpha|\mathcal{R}_{opt}|. \quad (4)$$

Note that $RCG(R, \mathcal{B}, \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{opt})$ is connected. Let $\mathcal{T}_{opt}$ be a minimum spanning tree of $RCG(R, \mathcal{B}, \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{opt})$. Let $\mathcal{T}_{min}$ be a minimum Steiner tree in $RCG(R, \mathcal{B}, \mathcal{X})$ which connects all nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$. Then we have

$$w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{min}) \leq w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}). \quad (5)$$

Since $\mathbb{B}$ is a β-approximation algorithm, we have

$$w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{\mathbb{B}}) \leq \beta w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{min}) \leq \beta w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}). \quad (6)$$

We can write $w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt})$ as $w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) = w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) + w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt})$, where $w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt})$ is the sum of the weights of the weight-1 edges in $\mathcal{T}_{opt}$ and $w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt})$ is the sum of the weights of the weight-2 edges in $\mathcal{T}_{opt}$. Let u be any node in $\mathcal{R}_{opt} \setminus \mathcal{R}_{\mathbb{A}}$. It follows from the geometric property of minimum spanning trees that u is incident with at most 5 weight-1 edges in $\mathcal{T}_{opt}$. Suppose, to the contrary, u is incident with 6 weight-1 edges (u, v_1), (u, v_2), (u, v_3), (u, v_4), (u, v_5), and (u, v_6). Then we have $\{v_1, v_2, \ldots, v_6\} \subseteq \mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$, and $d(u, v_i) \leq R$, $1 \leq i \leq 6$. Therefore there exist i and j ($1 \leq i < j \leq 6$) such that $d(v_i, v_j) \leq R$. Replacing the weight-1 edge (u, v_i) by the weight-0 edge (v_i, v_j) leads to a spanning tree of $RCG(R, \mathcal{B}, \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{opt})$ with a *smaller weight* than the weight of T_{opt}, which is a contradiction. Therefore we have

$$w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) \leq 5|\mathcal{R}_{opt}|. \quad (7)$$

Since $\mathcal{T}_{opt}$ is a tree, it has at most $|\mathcal{R}_{opt}| - 1$ weight-2 edges. Hence

$$w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) \leq 2(|\mathcal{R}_{opt}| - 1). \quad (8)$$

Therefore

$$w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) \leq 5|\mathcal{R}_{opt}| + 2(|\mathcal{R}_{opt}| - 1) = 7|\mathcal{R}_{opt}| - 2. \quad (9)$$

Combining (6), (9), and Lemma 3.1, we have

$$|\mathcal{R}_{\mathbb{B}}| \leq \frac{1}{2}w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{\mathbb{B}}) \leq \frac{\beta}{2}w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{T}_{opt}) \leq 3.5\beta|\mathcal{R}_{opt}|. \quad (10)$$

Combining (4) and (10), we have

$$|\mathcal{R}_{\mathbb{A},\mathbb{B}}| = |\mathcal{R}_{\mathbb{A}}| + |\mathcal{R}_{\mathbb{B}}| \leq (\alpha + 3.5\beta)|\mathcal{R}_{opt}|. \quad (11)$$

This proves the theorem. ■

We further remark on the implication of this framework. With different choices of algorithms $\mathbb{A}$ and $\mathbb{B}$, we will end up with approximation algorithms for 1CSCP with different approximation ratios and running times. For example, we can use the PTAS for GDHP of [26] as algorithm $\mathbb{A}$, and the $(1 + \frac{\ln 3}{2})$-approximation algorithm of [31] as algorithm $\mathbb{B}$. This combination leads to the following.

Corollary 4.1: The $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ problem has a polynomial time $(6.43 + \epsilon)$-approximation scheme. □

Though the above combination gives a good approximation ratio, the time complexity is high [7, 31]. In our numerical study, we used the 22-approximation algorithm for GDHP in [7] as algorithm $\mathbb{A}$, and the 2-approximation algorithm of [19] as algorithm $\mathbb{B}$. This combination leads to the following.

Corollary 4.2: The $1CSCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ problem has a 29-approximation algorithm with a running time of $\mathcal{O}(|\mathcal{X}|^2|\mathcal{S}|^4 + |\mathcal{B} \cup \mathcal{X}|^2 \log|\mathcal{B} \cup \mathcal{X}|)$. □

We note that an alternative approach to this problem could be to apply algorithm $\mathbb{B}$ on a subgraph of $HCG(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ without the SN-SN edges to obtain a connected single cover. However, since the number of SNs that are connected to an RN cannot be bounded by a constant, proving a meaningful approximation ratio for this approach is an open problem. Our best solution is the $(6.43 + \epsilon)$-approximation scheme.

5 Two-Connected Double-Cover

In this section, we study the 2-connected double-cover problem. In this problem, we strengthen the coverage requirement from single-cover to double-cover and the connectivity requirement from connected to 2-connected. Since the connected single-cover problem is NP-hard, we believe that the 2-connected double-cover problem is also NP-hard. Instead of concentrating on the NP-hardness proof, we focus on the design of efficient approximation algorithms for this problem. As in Section 3, let $\mathcal{B}$ be a set of base stations (BSs), $\mathcal{S}$ be a set of sensor nodes (SNs), and $\mathcal{X}$ be a set of candidate RN locations. Let $r > 0$ and $R \geq 2r$ be the communication range of the SNs, and that of the RNs, respectively.

5.1 Problem Definitions and Notation

Definition 5.1: [DCP] A subset $\mathcal{R} \subseteq \mathcal{X}$ is called a *double-cover* (denoted by DC) of $\mathcal{S}$ if every SN is within distance r of at least two nodes in $\mathcal{B} \cup \mathcal{R}$. The *size* of the corresponding DC is $|\mathcal{R}|$. A DC is called a *minimum double-cover* (denoted by MDC) of $\mathcal{S}$ if it has the minimum size among all DCs of $\mathcal{S}$. The *double-cover relay node placement problem* (denoted by DCP) for $(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ seeks an MDC of $\mathcal{S}$. □

Definition 5.2: [2CDCP] A subset $\mathcal{R} \subseteq \mathcal{X}$ is called a *2-connected double-cover* (denoted by 2CDC) of $\mathcal{S}$ if for every SN $s \in \mathcal{S}$ there exists a pair of node-disjoint paths from s to two base stations in the hybrid communication graph $HCG(r, R, \mathcal{B}, \mathcal{S}, \mathcal{R})$. The *size* of $\mathcal{R}$ is $|\mathcal{R}|$. $\mathcal{R}$ is called a *minimum 2-connected double-cover* (denoted by M2CDC) of $\mathcal{S}$ if it is a 2CDC of $\mathcal{S}$, and has the minimum size among all 2CDCs of $\mathcal{S}$. The 2-connected double-cover relay node placement problem for $(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ (denoted by 2CDCP) seeks an M2CDC of $\mathcal{S}$. □

The problem we are studying here is closely related to the $\{0, 1, 2\}$-*survivable network design problem* (SNDP) defined in the following.

Definition 5.3: [**$\{0, 1, 2\}$-SNDP**] Let $G = (V, E)$ be an undirected graph with nonnegative weights on all edges $e \in E$. For each pair of vertices $u, v \in V$, there is a connectivity requirement $c(u, v) \in \{0, 1, 2\}$. The $\{0, 1, 2\}$-*survivable network design problem* (SNDP) seeks a minimum weight subgraph $\mathcal{H}$ of G such that for any two vertices $u, v \in V$, $\mathcal{H}$ contains at least $c(u, v)$ node-disjoint paths between u and v. □

5.2 A Framework of Efficient Approximation Algorithms for 2CDCP

We present a general framework to solve 2CDCP, using an approximation algorithm $\mathbb{A}$ for DCP, and an approximation algorithm $\mathbb{B}$ for the $\{0, 1, 2\}$-SNDP problem. The framework of polynomial time approximation algorithms has an approximation ratio of $\alpha + 5\beta$, where α is the approximation ratio of $\mathbb{A}$, and β is the approximation ratio of $\mathbb{B}$. Our framework for 2CDCP is presented as Algorithm 2.

The major steps of our scheme are as follows. First, we apply the approximation algorithm $\mathbb{A}$ to obtain a double cover $\mathcal{R}_{\mathbb{A}} \subseteq \mathcal{X}$ of $\mathcal{S}$. This is accomplished in Line 1 of the algorithm. In Line 2, we construct the relay communication graph $G = RCG(R, \mathcal{B}, \mathcal{X})$.

Algorithm 2 Approximation for 2CDCP$(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$

Input: Set $\mathcal{B}$ of BSs, set $\mathcal{S}$ of SNs, set $\mathcal{X}$ of candidate RN locations, sensor node communication range $r > 0$, and relay node communication range $R \geq 2r > 0$. An approximation algorithm $\mathbb{A}$ for DCP, and an approximation algorithm $\mathbb{B}$ for the $\{0, 1, 2\}$-$SNDP$ problem.

Output: A connected double-cover $\mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{\mathbb{B}} \subseteq \mathcal{X}$.

1: Apply algorithm $\mathbb{A}$ to obtain a double-cover $\mathcal{R}_{\mathbb{A}}$ of $\mathcal{S}$. Without loss of generality, we assume that $\mathcal{R}_{\mathbb{A}}$ is *minimal*, meaning that none of its proper subsets is a double-cover of $\mathcal{S}$.

2: Construct the relay communication graph $G = RCG(R, \mathcal{B}, \mathcal{X})$.

3: **if** the nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$ are not in the same 2-connected component of $RCG(R, \mathcal{B}, \mathcal{X})$ **then**

4:   Stop. The given instance does not have a 2-connected double-cover.

5: **end if**

6: Assign *edge weight* in G as in (1), with $\mathcal{R}$ substituted by $\mathcal{R}_{\mathbb{A}}$, i.e., for an edge (x, y) in G, we have that

$$w^{\mathcal{R}_{\mathbb{A}}}(x, y) = |(\mathcal{X} \setminus \mathcal{R}_{\mathbb{A}}) \cap \{x, y\}|.$$

For a node x and a node y ((x, y) does not have to be an edge), assign the connectivity requirement to be $c(x, y) = 2 - |(\mathcal{X} \setminus \mathcal{R}_{\mathbb{A}}) \cap \{x, y\}|$.

7: Apply algorithm $\mathbb{B}$ to compute a low weight 2-connected subgraph $\mathcal{H}_{\mathbb{B}}$ of $RCG(R, \mathcal{B}, \mathcal{X})$, spanning the node set $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$. Let the set of added relay nodes in the solution to $\mathbb{B}$ be $\mathcal{R}_{\mathbb{B}}$.

8: Output $\mathcal{R}_{\mathbb{A},\mathbb{B}} = \mathcal{R}_{\mathbb{A}} \cup \mathcal{R}_{\mathbb{B}}$.

The given instance of the problem has a feasible solution if and only if all the nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$ are in the same 2-connected component of G. Lines 3 to 5 check whether there exists a 2-connected component containing all the nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$. If there is no such 2-connected component, the algorithm stops. In Line 6, we assign non-negative edge weights to the edges in G according to (1). Then, in Line 7, we apply algorithm $\mathbb{B}$ to compute a low cost 2-connected subgraph of G spanning all the nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$. Finally, Line 8 outputs the locations to place the RNs.

Theorem 5.1: The worst case running time of Algorithm 2 is $O(T_{\mathbb{A}} + T_{\mathbb{B}} + |\mathcal{B} \cup \mathcal{X}|^2)$, where $T_{\mathbb{A}}$ and $T_{\mathbb{B}}$ are the time complexities of the approximation algorithms $\mathbb{A}$ and $\mathbb{B}$, respectively. Furthermore, we have:

(a) $2CDCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ has a feasible solution if and only if $\mathcal{S}$ has a double-cover $\mathcal{R} \subseteq \mathcal{X}$ such that all nodes in $\mathcal{B} \cup \mathcal{R}$ are in the same 2-connected component of $RCG(R, \mathcal{B}, \mathcal{X})$.

(b) When $2CDCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$ has a feasible solution, Algorithm 2 guarantees computing a 2-connected double-cover of $\mathcal{R}_{\mathbb{A},\mathbb{B}}$ of $\mathcal{S}$, which uses no more than $(\alpha + 5\beta)$ times the number of

RNs required in an optimal solution $\mathcal{R}_{opt}$ to $2CDCP(r, R, \mathcal{B}, \mathcal{S}, \mathcal{X})$, where α and β are the approximation ratios of $\mathbb{A}$ and $\mathbb{B}$, respectively. □

Before proceeding to prove Theorem 5.1, we need the following lemma, which will be used to relate $|\mathcal{R}_{\mathbb{B}}|$ to the number of RNs in the optimal solution.

Lemma 5.1: Let $\mathcal{R}$ be a 2-connected double-cover of $\mathcal{S}$. Let $\mathcal{R}_c$ be a subset of $\mathcal{R}$ such that $\mathcal{R}_c$ is a double-cover of $\mathcal{S}$. Let $\mathcal{R}_n = \mathcal{R} \setminus \mathcal{R}_c$. Assign edge weight in $RCG(R, \mathcal{B}, \mathcal{R})$ such that the weight of edge (x, y) is $w^{\mathcal{R}_c}(x, y) = |\mathcal{R}_n \cap \{x, y\}|$. Let $\mathcal{H}_{\mathcal{R}_c,\mathcal{R}_n}$ be a minimum 2-connected subgraph connecting all nodes in $\mathcal{B} \cup \mathcal{R}$. Then, for each node $u \in \mathcal{R}_n$, u is connected to at most 5 nodes in $\mathcal{R}_c$, and at most one node in $\mathcal{B}$. □

PROOF. For the proof, we refer the reader to the Lemma 4.1 in [25], while noting the following differences. The HCG there corresponds to the RCG here, with $\mathcal{S}$ there corresponding to $\mathcal{R}$ here, and $\mathcal{R}$ there corresponding to $\mathcal{R}_{opt} \setminus \mathcal{R}$ here. ■

PROOF OF THEOREM 5.1: The proofs of the running time and Part (a) are similar to those of Theorem 4.2. Here we concentrate on the approximation ratio.

Roadmap. Following the same logic as in the proof of Theorem 4.2, we prove the ratio by proving $|\mathcal{R}_{\mathbb{A}}| \leq \alpha|\mathcal{R}_{opt}|$ and $|\mathcal{R}_{\mathbb{B}}| \leq 5\beta|\mathcal{R}_{opt}|$, respectively. The ratio then follows from the fact that $|\mathcal{R}_{\mathbb{A},\mathbb{B}}| = |\mathcal{R}_{\mathbb{A}}| + |\mathcal{R}_{\mathbb{B}}|$.

Recall that the set of RNs required by $\mathbb{A}$ is $\mathcal{R}_{\mathbb{A}}$ and the set of extra RNs required by algorithm $\mathbb{B}$ is $\mathcal{R}_{\mathbb{B}}$.

Let $\mathcal{R}_{opt}$ be an optimal solution to 2CDCP and $\mathcal{R}_{mdc}$ be an optimal solution to DCP. It is clear that $|\mathcal{R}_{mdc}| \leq |\mathcal{R}_{opt}|$. As $\mathbb{A}$ has approximation ratio α, we have

$$|\mathcal{R}_{\mathbb{A}}| \leq \alpha|\mathcal{R}_{mdc}| \leq \alpha|\mathcal{R}_{opt}|. \tag{12}$$

Let $\mathcal{H}_{min}$ be an optimal solution to the $\{0, 1, 2\}$-SNDP instance that connects all nodes in $\mathcal{B} \cup \mathcal{R}_{\mathbb{A}}$, and $\mathcal{H}_{opt}$ be a solution that uses the optimal number of RNs required to solve 2CDCP. Since $\mathcal{H}_{opt}$ is a feasible solution to $\{0, 1, 2\}$-SNDP, and $\mathbb{B}$ is a β-approximation algorithm for $\{0, 1, 2\}$-SNDP, we have

$$w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{\mathbb{B}}) \leq \beta \cdot w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{min}) \leq \beta \cdot w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}). \tag{13}$$

We need to find an upper bound on $w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt})$ using a function of $|\mathcal{R}_{opt}|$. Let $w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt})$ denote the total weights of the weight-2 edges in $\mathcal{H}_{opt}$, and let $w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt})$ denote the total weights of the weight-1 edges in $\mathcal{H}_{opt}$. We have

$$w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}) = w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}) + w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}). \tag{14}$$

Let each RN in $\mathcal{H}_{opt}$ be incident with at most $\Delta(\mathcal{H}_{opt})$ weight-1 edges in $\mathcal{H}_{opt}$, we have

$$w_1^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}) \leq |\mathcal{R}_{opt}| \cdot \Delta(\mathcal{H}_{opt}). \tag{15}$$

Applying Lemma 3.3 to each of the connected components of the subgraph of $\mathcal{H}_{opt}$ induced by all the 2-weight edges, we have

$$w_2^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}) \leq 2 \cdot (2|\mathcal{R}_{opt}| - 1). \tag{16}$$

It follows from Lemma 3.1 that

$$\begin{aligned} |\mathcal{R}_{\mathbb{B}}| = \pi^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{\mathbb{B}}) &\leq \frac{1}{2} w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{\mathbb{B}}) & (17)\\ &\leq \frac{\beta}{2} w^{\mathcal{R}_{\mathbb{A}}}(\mathcal{H}_{opt}) & (18)\\ &\leq \frac{\beta}{2}(4 + \Delta(\mathcal{H}_{opt}))|\mathcal{R}_{opt}|. & (19)\end{aligned}$$

It follows from Lemma 5.1 that

$$|\mathcal{R}_{\mathbb{B}}| \leq \frac{\beta}{2}(4 + 6)|\mathcal{R}_{opt}| \leq 5\beta|\mathcal{R}_{opt}|. \tag{20}$$

Combining (12) and (20), we have

$$|\mathcal{R}_{\mathbb{A},\mathbb{B}}| = |\mathcal{R}_{\mathbb{A}}| + |\mathcal{R}_{\mathbb{B}}| \leq (\alpha + 5\beta)|\mathcal{R}_{opt}|. \tag{21}$$

This proves the theorem. ■

We can use different combinations of $\mathbb{A}$ and $\mathbb{B}$ for different purposes. Two examples are summarized in the following two corollaries.

Corollary 5.1: The $2CDCP$ problem has a $\mathcal{O}(\ln \mathbf{n})$-approximation algorithm with a polynomial running time. □

PROOF: This is achieved by choosing $\mathbb{A}$ as the $\mathcal{O}(\ln n)$-approximation algorithm by Rajagopalan *et al.* [28] and $\mathbb{B}$ as the 3-approximation algorithm of Ravi *et al.* for the $\{0, 1, 2\}$-SNDP problem [29, 30]. ■

In the study above, we have made no assumption on the number of RNs that an SN can be connected to. However, in many practical scenarios, the number of RNs that an SN can be connected to is usually bounded by a constant f. In this case, a simple modification to the constant frequency based algorithm in [15] will result in a constant-factor approximation algorithm as stated in the following corollary.

Corollary 5.2: If the number of RNs that can be connected to an SN is bounded by a constant f, then Algorithm 2 guarantees a feasible solution in polynomial time, with a constant approximation ratio $f + 5\beta$, where β is the approximation ratio of $\mathbb{B}$. □

6 COMPUTATION OF LOWER BOUNDS

No algorithms have been developed for solving the problem of two-tiered constrained relay node placement in the existing literature, because of its difficulty. Hence due to the lack of a more suitable comparison, we compare the results from our algorithms with that obtained from Linear Program (LP) formulations of 1CSCP and 2CDCP. We note that an LP can be solved in polynomial time. Also, the solution to the LP version of the problem denoted by $\mathcal{R}_{LP}$, is a lower bound of the solution to the corresponding Integer Linear Program (ILP) version. That is, $|\mathcal{R}_{LP}| \leq |\mathcal{R}_{opt}|$. The suitability of the use of LP for comparison comes from two facts. On one hand, if $\mathbb{A}$ is an α-approximation algorithm for a problem, then $|\mathcal{R}_{\mathbb{A}}| \leq \alpha \cdot |\mathcal{R}_{LP}|$, consequently, $|\mathcal{R}_{\mathbb{A}}| \leq \alpha \cdot |\mathcal{R}_{opt}|$. On the other hand, solving the LP may take much less time in comparison to solving the corresponding ILP, which may have an exponential running time.

Closely following the LP formulation in [25], we formulate the LPs for 1CSCP and 2CDCP as multi-commodity flow problems, with a flow originating from each SN u (the source) to a fictitious sink d, which is connected to all BSs. The value of the flow originating from the source is designated as $\mathbf{f}$. For 1CSCP, $\mathbf{f} = 1$ and for 2CDCP $\mathbf{f} = 2$.

$$\min \sum_{j=1}^{|\mathcal{X}|} x_j \tag{22}$$

subject to,

Requirements Constraints:

$$x_j - \gamma_{ij} \geq 0, \forall j \in \mathcal{X}, \forall i \in \mathcal{S} \tag{23}$$

Source Constraints:

$$\sum_{(i,j)\in HCG} f_{iji} - \sum_{(j,i)\in HCG} f_{jii} = \mathbf{f}, \forall i \in \mathcal{S} \tag{24}$$

Sink Constraints:

$$\sum_{j\in\mathcal{B}} f_{jdi} - \sum_{j\in\mathcal{B}} f_{dji} = \mathbf{f}, \forall i \in \mathcal{S} \tag{25}$$

Flow satisfaction constraints:

$$\sum_{(i,j)\in HCG} f_{ijk} - \gamma_{kj} = 0, \forall k \in \mathcal{S}, \forall j \in \mathcal{X} \tag{26}$$

Flow conservation constraints:

$$\sum_{(i,j)\in HCG} f_{ijk} \quad - \sum_{(j,i)\in HCG} f_{jik} = 0, \forall j \in \mathcal{X}, \forall k \in \mathcal{S} \tag{27}$$

$$\sum_{(i,j)\in HCG\cup(d,j)} f_{ijk} \quad - \sum_{(j,i)\in HCG\cup(j,d)} f_{jik} = 0, \forall j \in \mathcal{B}, \forall k \in \mathcal{S} \tag{28}$$

Bounds:

$$f_{ijk} = 0, \forall i, j \in \mathcal{B}, \forall k \in \mathcal{S} \tag{29}$$

$$0 \leq \gamma_{ij}, f_{ijk} \leq 1, \forall \gamma_{ij}, \text{remaining } f_{ijk}'s. \tag{30}$$

The variable $x_j, j = 1, \ldots, |\mathcal{X}|$, is the flow variable, which represents the maximum flow of any commodity through the RN j for the flow problem to be satisfied. It may be regarded as the fraction of the RN j that is used in solving the relay node placement problem. The variable $\gamma_{ij}, i = 1, \ldots, |\mathcal{S}|, j = 1, \ldots, |\mathcal{X}|$, is the requirement variable. It represents the amount of flow from an SN i required to pass through an RN j. To ensure that the coverage and/or connectivity requirements are satisfied, the flow variables for each RN have to satisfy the requirements constraints given by Equation (23). The variable f_{ijk} represents the flow of commodity k from node i to j. Equations (24) and (25) represent the source and the sink constraints respectively. The source flow constraints are easy to understand. The sink constraints are specified with respect to the fictitious sink which is connected to the BSs only. The flow satisfaction constraints, given by Equation (26), enforce that the amount of flow of commodity k flowing into an RN j from nodes adjacent to it is exactly equal to the amount of flow of k that is required to flow through j. That is, j does not create nor buffer any flow. Equations (27) and (28) represent the flow conservation at the RNs and the BSs respectively. Corresponding to each BS there is an additional edge connecting it to d for the flows to reach the destination. Equation (29) represents the bounds for the flow variables corresponding to the flow between the BSs. These flow variables are set to zero to ensure that each BS can only appear in at most one path from a source to d.

7 Experimental Results

In this section, we evaluate the performance of our algorithms through extensive experiments.

7.1 Network Setup

As in [18], [25] and [36], the set $\mathcal{S}$ of SNs were randomly distributed in a square region. Two base stations were deployed randomly in the region. For transmission ranges, we set $r = 15$ and $R = 30$. For the candidate RN locations, we considered two different distributions: *grid distribution* and *random distribution*. For grid distribution, the deployment region consists of $K \times K$ small squares each of side 10, with the candidate RN locations being the $(K+1)^2$ grid points. For random distribution, the candidate RN locations were *randomly* distributed in the deployment region.

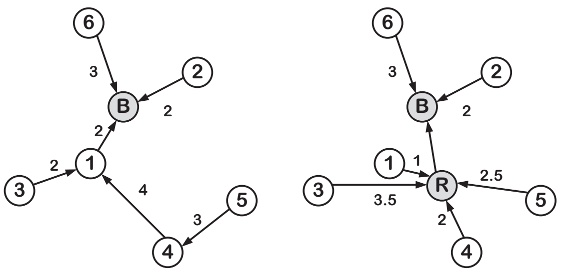
We report two separate deployments of the SNs: the case where the density of the SNs in the region increases, and the case where the density is constant. We define the density as the ratio of the number of SNs in the region to the area of the region. For the increasing density case, we chose a constant region of size 100×100 sq. units. The number of candidate RN locations was $(K + 1)^2 = 121$, while the number of SNs was varied from 20 to 120 in increments of 20. For the constant density case, with the increase in the number of SNs the size of the region increased. We studied two sub-cases: density $d_1 = 0.005$ and density $d_2 = 0.01$. For each density value, we used 7 different numbers of SNs. The deployment region sizes were chosen to be $40 \times 40, 50 \times 50, \ldots, 100 \times 100$, with the number of SNs ranging from 8 to 50 for d_1, and 16 to 100 for d_2. For each setting the results were averaged over 10 test cases.

7.2 Algorithm Selection

To evaluate the effectiveness of our frameworks, we implemented Algorithm 1 and Algorithm 2. For Algorithm 1, we used the 22-approximation algorithm in [7] as $\mathbb{A}$ and the 2-approximation algorithm in [19] as $\mathbb{B}$. Our choices were motivated by their faster running times. The numerical results show that the solutions produced are still within twice the optimum for all of our test cases. We use ACS to denote this implementation of Algorithm 1. For Algorithm 2, we used the 3-approximation algorithm in [29] as $\mathbb{B}$. We implemented two choices of $\mathbb{A}$ for the set multicover problem: the greedy $\mathcal{O}(\ln n)$-approximation algorithm in [28] (we use ALD to denote this combination), and the frequency-based algorithm in [15] (we use ACD to denote this combination). To ensure rigorous comparison, for each instance of 1CSCP and 2CDCP problems that were solved by our algorithms, we also solved the corresponding LP instance (for lower bound) and evaluated the proximity of our solutions to the optimal. We denote the solutions for 1CSCP and 2CDCP as LPS and LPD respectively.

7.3 Performance Metrics

The performance metrics include *the running time*, *the number of RNs used* and *Energy Efficiency*. For simplicity, we only report energy consumption for 1CSCP. To measure energy cost on each SN, we assume the following energy model. Assume the routing topology is given by the $|\mathcal{B}|$ trees rooted at the BSs which are induced from the Steiner tree computed. For each SN s_i, let $c(s_i)$ denote the number of s_i's children in the tree. Let $p(s_i)$ denote s_i's parent. We assume that the energy consumption is an exponential function of the transmission distance. Therefore the energy cost of s_i is $e(s_i) = ||p(s_i), s_i||^\gamma \cdot (c(s_i)+1)$. Intuitively, the energy cost of s_i is equal to the product of the transmission power required to transmit to the next hop along the path to the BS and the number of SNs whose paths pass through it (including itself). Throughout our the experiments, we assume that $\gamma = 2$. We define the *energy cost* of the network as the maximum energy consumption among all the SNs, because such a sensor determines the lifetime of the network.

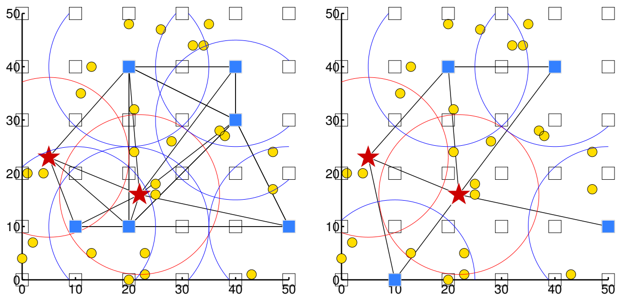


(a) Topology without relay node placement (b) Topology with relay node placement

Fig. 3: Network topology with or without relay node placement. For (a), the energy cost is $4^2 \times 2 = 32$, which occurs at s_4. For (b), the energy cost is $3.5^2 \times 1 = 12.25$, which occurs at s_3.

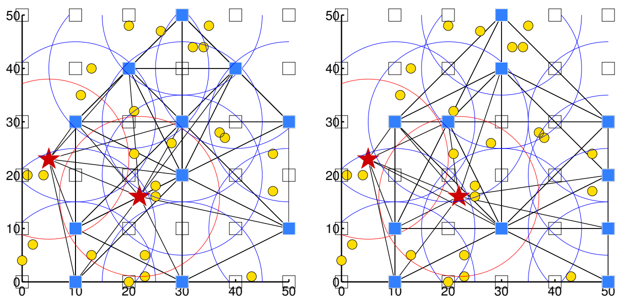
Without the relay node placement, we assume that the routing topology is induced from a minimum spanning tree, where the weight between SN and SN or between SN and BS is set to the Euclidean distance between them, and the weight between a pair of BSs is set to 0. An example is shown in Fig. 3(a). With the relay node placement, each SN transmits to the closest RN or BS directly. The energy cost can be computed similar as above. A corresponding example is shown in Fig. 3(b).

7.4 Illustration of Relay Node Placement

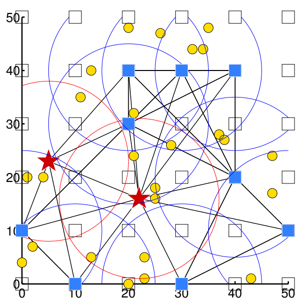


(a) ACS (b) OPTS

Fig. 4: RN placement for 1CSCP: grid distribution. The numbers of RNs used by ACS and OPTS are 6 and 4, respectively.



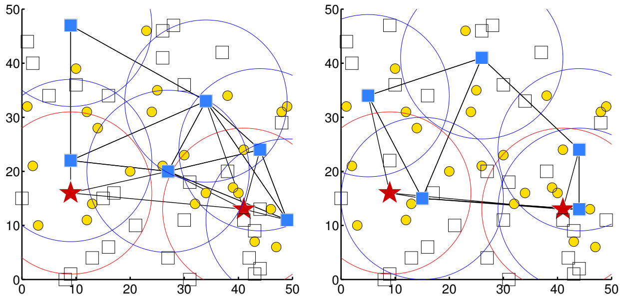
(a) ALD (b) ACD



(c) OPTD

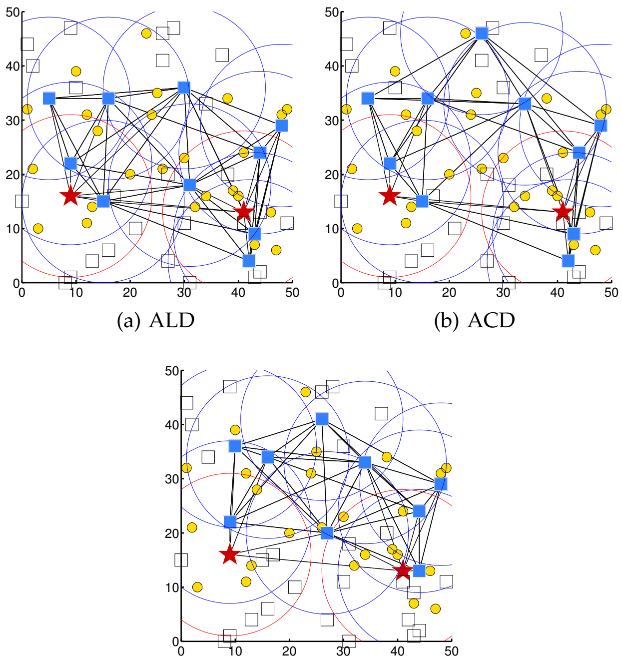
Fig. 5: RN placement for 2CDCP: grid distribution. The numbers of RNs used by ALD, ACD and OPTD are 11, 11 and 9, respectively.

In this section, we illustrate the relay node placement obtained by different algorithms through Fig. 4 to Fig. 7. Here, we solve 1CSCP and 2CDCP optimally using the LP formulation in Section 6 with integer constraints, i.e., the ILP formulation. We denote the optimal algorithms for 1CSCP and 2CDCP by $OPTS$ and $OPTD$, respectively. In each figure, the red stars represent the BSs, the yellow disks represent the SNs



(a) ACS (b) OPTS

Fig. 6: RN placement for 1CSCP: random distribution. The numbers of RNs used by ACS and OPTS are 6 and 5, respectively.



(a) ALD (b) ACD

(c) OPTD

Fig. 7: RN placement for 2CDCP: random distribution. The numbers of RNs used by ALD, ACD and OPTD are 10, 10 and 9, respectively.

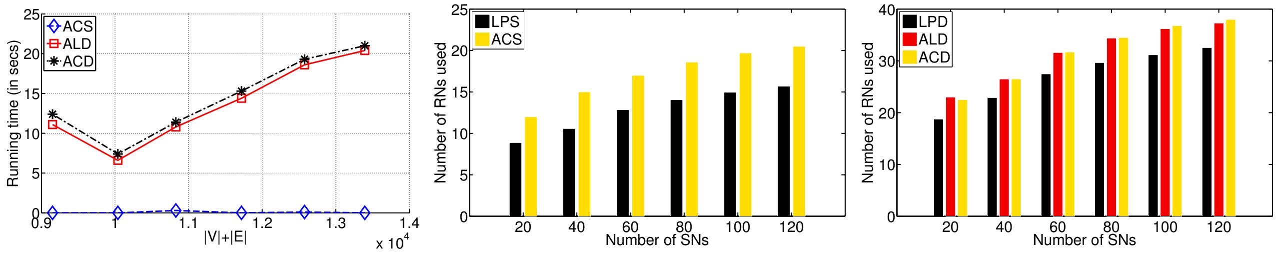
and the blue squares represent the RNs, of which the solid ones represent the deployed RNs. The circle centered at the BS or the RN is of radius r. Therefore a SN within a circle can transmit to the corresponding BS or RN. The link between RN and RN or between RN and BS indicates that these two nodes are within the transmission range of each other. For all the examples, the region size is 50×50. There are 2 BSs, 25 SNs and 36 candidate RN locations distributed in the deployment region.

7.5 Result Analysis

For the case of increasing SN density, Fig. 8(a) and Fig. 9(a) show the running time of ACS, ALD, and ACD in the network with the grid RN distribution and the random RN distribution, respectively. Since the running time of the algorithms is based on the number of edges ($|E|$) and the number of vertices ($|V|$), the X-axis represents the average of $|V| + |E|$, while the Y-axis represents the running time in seconds. The dashed blue line (diamond markers) shows the running time of ACS. The solid red line (square markers) and the dash-dot black line (star markers) show the running time of ALD and ACD respectively. In case of ALD and ACD, the running time decreases at first and then increases for the grid RN distribution. This is because, for the initial data point (20 SNs), the RNs chosen as a cover are not connected, and connecting them with additional RNs increases the running time. However, with an increase in the number of SNs the number of RNs needed for a cover becomes adequate to form a connected network by themselves. With increasing number of SNs the running time increases as it takes more time to find a cover. This also holds true for ACS. However, since ACS is much faster than the other algorithms, it is not obvious in the figure. The bigger dip and faster increase of the running time of ALD and ACD (for 2CDCP) in comparison to that of ACS (for 1CSCP) is because of the use of the higher complexity SNDP algorithm. For the random RN distribution case, we do not observe the V-shape in the running time curve. The reason is that, unlike the grid distribution, where the RNs are fixed at the grid points, the RNs in the random distribution case are randomly distributed in the region. It is highly likely to have a connected cover after the first phase of the algorithm.

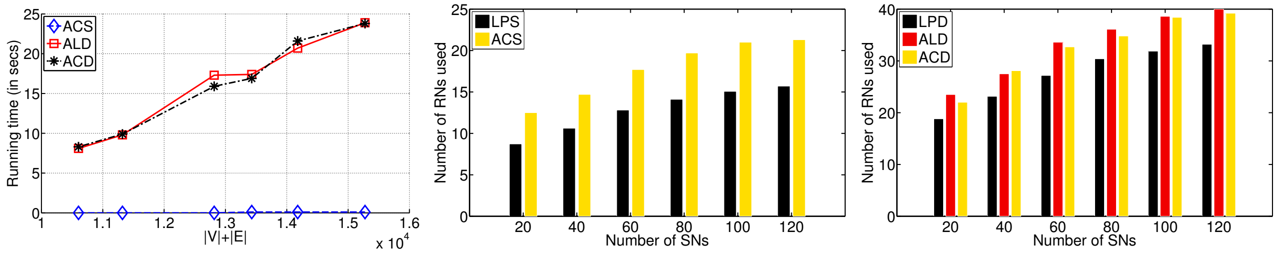
For the grid RN distribution case, Figs. 8(b) and 8(c) show the average number of RNs required by ACS and LPS, for solving 1CSCP, and by ALD, ACD, and LPD, for solving 2CDCP respectively. For the random RN distribution case, the corresponding results are shown in Figs. 9(b) and 9(c). Both approximation algorithms perform pretty well in comparison to the lower bound. The number of RNs obtained is never more than *twice* the cost of the LP solution. Thus the results from our approximation algorithms are *never more* than twice the optimal value. This indicates that our approximation algorithms perform very well. We note that for 2CDCP, both the approximation algorithms have the same results despite the disparities in their approximation ratios. This is because, the pathological cases where the $\mathcal{O}(\ln n)$ algorithm performs badly do not occur in the square grid with random SNs deployment. We will study this in the future.

For the case of constant density, we studied two sub-cases: density $d_1 = 0.005$ and density $d_2 = 0.01$. For each density value, we used 7 different numbers of SNs. The deployment region sizes were chosen to be $40 \times 40, \ldots, 100 \times 100$, with the number of SNs ranging from 8 to 50 for d_1, and 16 to 100 for d_2. Fig. 10(a) and Fig. 11(a) show the running times of ACS, ALD, and ACD for both the densities. For both



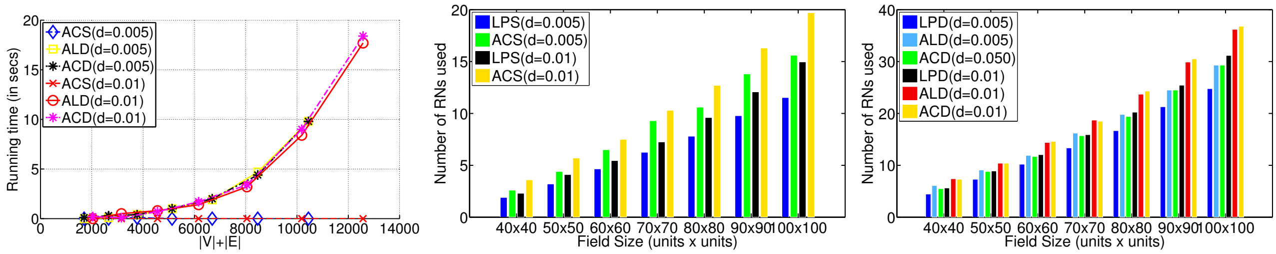
(a) running times of ACS, ALD, ACD (b) #RNs used by ACS, LPS (c) #RNs used by ALD, ACD, LPD

Fig. 8: Results for grid RN distribution with increasing SN density: 100×100 deployment region; $|\mathcal{X}| = 121$; $|\mathcal{S}| = 20, 40, 60, 80, 100, 120$.



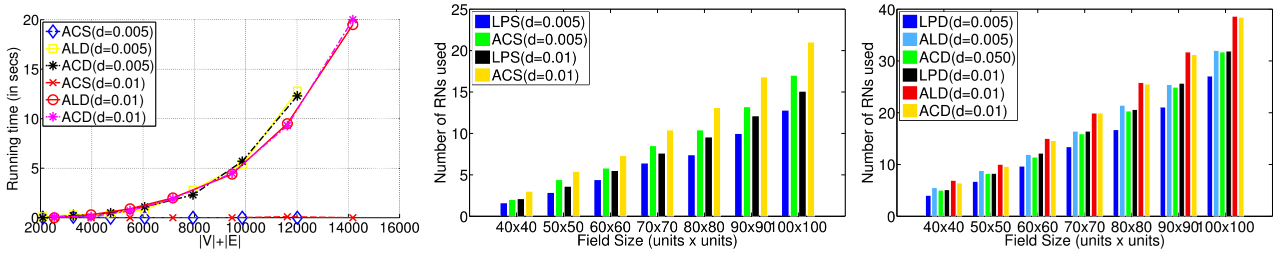
(a) running times of ACS, ALD, ACD (b) #RNs used by ACS, LPS (c) #RNs used by ALD, ACD, LPD

Fig. 9: Results for random RN distribution with increasing SN density: 100×100 deployment region; $|\mathcal{X}| = 121$; $|\mathcal{S}| = 20, 40, 60, 80, 100, 120$.



(a) running times of ACS, ALD, ACD (b) #RNs used by ACS, LPS (c) #RNs used by ALD, ACD, LPD

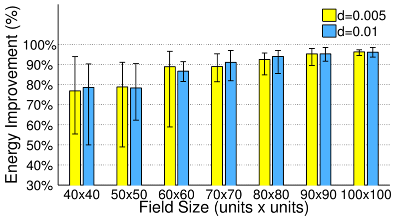
Fig. 10: Results for grid RN distribution with constant SN density: seven different deployment regions, from 40×40 to 100×100; two density values, $d_1 = 0.005$ and $d_2 = 0.01$.



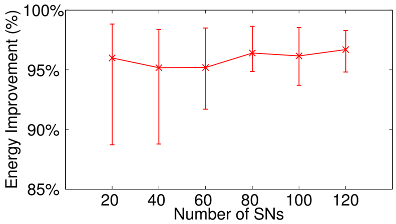
(a) running times of ACS, ALD, ACD (b) #RNs used by ACS, LPS (c) #RNs used by ALD, ACD, LPD

Fig. 11: Results for random RN distribution with constant SN density: seven different deployment regions, from 40×40 to 100×100; two density values, $d_1 = 0.005$ and $d_2 = 0.01$.

densities, the algorithms for 1CSCP have low running time, while those for 2CDCP have higher running time, which is expected. The running times for the ALD and ACD, increase with increasing $|E| + |V|$ as more RNs are required for coverage and connectivity, thus taking more time. The running time for $d_2 = 0.01$ is less than that of $d_1 = 0.005$, as with increasing density, there is greater likelihood of the RNs used for coverage to be connected as such. Figs. 10(b), 10(c), 11(b), and 11(c) show the number of RNs required by various algorithms in different experiment settings. Again, our algorithms perform remarkably well, never requiring more than twice the number of RNs in the optimal solution.

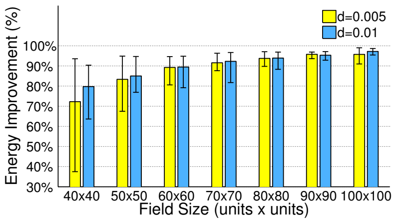


(a) Constant density

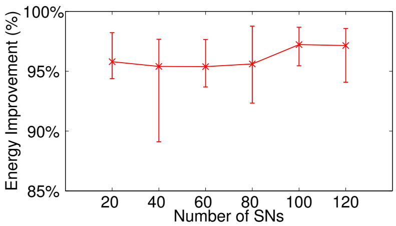


(b) Increasing density

Fig. 12: Grid distribution



(a) Constant density



(b) Increasing density

Fig. 13: Random distribution

For energy efficiency, we only compare the network without RNs deployed and the network with RNs deployed to solve 1CSCP. Figs. 12 and 13 show the energy efficiency improvement by placing RNs. For the constant density case, we observe that the improvement is increased with the increase of the field size. The reason is that some SNs may need to forward data for more SNs when the field size is increased without RNs deployed. In the case with RNs deployed, the SNs can transmit to the closest RN and do not need to forward data for other SNs. For the increasing density case, we observe that the improvement tends to keep consistent (the average improvement varies within the range of width 3%). The reason is that although the SNs need to forward the data for more SNs, the transmission distances are shorten due to the increased SN density.

8 Conclusions and Open Problems

We have studied the constrained relay node placement in a two-tiered wireless sensor network to meet connectivity and survivability requirements. For the connected single-cover problem, we have presented a framework of polynomial time approximation algorithms with $\mathcal{O}(1)$-approximation ratios. For the 2-connected double-cover problem, our algorithms have $\mathcal{O}(1)$-approximation ratios for the practical cases where each sensor node can be connected to only a constant number of relay nodes, and $\mathcal{O}(\ln n)$-approximation ratios for the arbitrary cases. We have also presented linear programming based lower bounds for the optimal solutions, which are used in the simulation studies. Simulation results show that the solutions obtained by our algorithms are always within twice that of the optimal solution.

There are still many open problems in this area. Although algorithms with guaranteed approximation ratios were developed in this paper, one underlying assumption in our model is that $R \geq 2r$. It would be interesting to study the two-tiered constrained relay node placement problem without such an assumption. In addition, we have been tackling the relay node placement problems by solving the coverage problem and the connectivity problem separately. Alternatively, one can develop an algorithm that considers both coverage and connectivity jointly, which may lead to a tighter bound. Another direction is to extend our algorithms to more generic case, i.e., k-connected m-cover problem, where $k \geq 1$ and $m \geq 1$.

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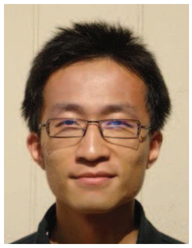

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